Innovation, International Trade, and Structural Change

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Abstract

Traditional theories of structural transformation fail to account for the disparities between employment and value added shares, which poses a significant puzzle. To address this issue, I propose a Schumpeterian framework, incorporating technological innovation and trade at the sector level. This framework makes distinct predictions regarding employment and value added shares. In a closed economy, the model establishes an equilibrium where the share of value added equals the share of employment. However, when a country opens up to trade and achieves a monopoly through innovation in a specific sector, it results in higher profits and greater value added relative to employment in that sector. Consequently, the share of value added increases more rapidly than the share of labor. Conversely, in sectors where the country lacks global monopolistic control, the share of value added diminishes due to lower profits for intermediate good producers, resulting in a value added share that is lower than the employment share.

KEYWORDS: Schumpeterian growth, Structural change, Innovation, Trade

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1 Introduction

A central concept in development economics is the notion of structural change, which is defined as the reallocation of economic resources across sectors with different productivity levels. Therefore, the three most common measures of economic structural change are sectoral employment shares, sectoral value added shares, and sectoral final consumption expenditure shares. While the literature often treats these measures as interchangeable, they are quantitatively distinct. Herrendorf et al. (2014) document such differences and Kuznets (1967) demonstrated that during the early stages of US development, the employment share of services increased significantly, while the value added share of services remained relatively constant. Furthermore, Buera & Kaboski (2009) consider this discrepancy to be a relevant puzzle for the theories of structural transformation for several countries.

This paper proposes a new theory whereby changes in innovations across different sectors over time account for the divergent paths in employment shares and value added shares. Indeed, Herrendorf et al. (2014) have pointed to international trade dynamics and variations in measurement methodologies as factors contributing to disparities between production and consumption measures[1] of economic structural transformation. Specifically, by taking into account international trade, where a portion of domestic consumption is sourced from abroad rather than domestically produced, differences between production and consumption measures of structural change can be elucidated. Additionally, differences in these production and consumption measures may manifest through the divergence in perspectives and methodologies utilized in economic accounting to assess economic activity, particularly concerning final consumption expenditure and value added in production. However, traditional theories of structural transformation cannot explain the differences between employment and value added shares. A notable puzzle arises from the incongruity between sectoral employment shares and sectoral value added shares, both of which are production measures.

In this paper, I consider a Schumpeterian paradigm of structural transformation in which technological innovation serves as the principal driver of sectoral productivity growth. There are three final good sectors that competitively produce three distinct final goods using labor and a continuum of intermediate goods. Importantly, only intermediate goods, which are produced monopolistically across all three sectors, are eligible for trade. The theoretical trade model is built on Aghion & Howitt (2009) and considers two countries, "home" and "foreign". The range of intermediate goods in each country is identical, and all countries produce exactly the same final products: foods, manufacturing goods, and services. Within each intermediate sector the world market can then be monopolized by the producer with the lowest cost and holding a patent for the most recent version of the intermediate good. To incorporate the demand side of structural change, I adopt the approach proposed by Comin et al. (2021), wherein Constant Elasticity of Substitution (CES) nonhomothetic preferences for households are introduced. Indeed, CES nonhomothetic preferences possess favorable properties for examining long-run structural change as they differentiate the impact of income on the growth of luxury goods sectors.

The closed economic model predicts that the share of value-added equals the share of employment. This relationship is based on the direct correlation between profits generated within a sector and the level of employment in that sector. As employment increases within a sector, both output and demand for intermediate goods also increase, leading to higher profits for monopolist producers of intermediate goods. Consequently, the income of both workers and entrepreneurs follows a linear pattern determined by the wage rate and the level of employment within sectors

[1] The employment shares and value added shares are related to production whereas final consumption expenditure shares are related to consumption.

utilizing intermediate goods.

However, once the country opens up to international trade, sectors in which the country becomes a global monopolist experience an increase in profit. This profit is now dependent on both domestic employment and employment from the rest of the world in the same sector. Thus, domestic monopolists benefit from a surplus of profit due to external demand for the latest versions of sectoral intermediate goods. Since a portion of the total profit is attributable to external demand, the share of value added in this sector is higher than that of employment. Conversely, in sectors where the country is not the global monopolist and therefore imports some intermediate goods, profit is lower than in a closed economy, and the share of employment is higher than that of value added. It is important to note that while international trade can explain disparities between consumption and production expenditures, it is insufficient in explaining the disparity between value-added and employment shares. However, by considering monopolistic rights that guarantee additional profits solely linked to an increase in demand, we can enhance our understanding of this disparity.

The literature on structural transformation in economics has witnessed significant evolution, with seminal contributions from Lewis (1954), Cheneryr (1960), Kuznets (1967), Baumol (1967), and Harris & Todaro (1970). These works collectively lay the groundwork for understanding the process of structural transformation, emphasizing key factors such as surplus labor, industrialization, sequential stages of growth, income inequality dynamics, the "cost disease" phenomenon, and rural-urban migration patterns. They have provided valuable insights into the drivers and consequences of structural change, guiding research and policy discussions on economic development and inequality mitigation. Building upon these foundational works, Ngai & Pissarides (2007) formalized Baumol's price effect by showing that different sectoral productivity growth rates account for shifts in sector final goods prices and demands for different goods.

Furthermore, the role of international trade in influencing structural transformation has received considerable attention, with studies by Matsuyama (2009) and Uy et al. (2013) exploring the connections between trade openness, sectoral specialization, and employment patterns[2]. Despite the progress made, challenges remain in reconciling disparities between employment and value-added shares across sectors, as highlighted by Herrendorf et al. (2014), while Buera & Kaboski (2009) identified that the behavior of consumption and output shares differs significantly from that of employment shares. They argued that models incorporating home production, sector-specific factor distortions, and differences across sectors in the accumulation of human capital are promising avenues to amend standard models. Additionally, Saenz (2020) showed that considering time-varying capital intensities can account for differences between employment and value added for South Korea.

This paper contributes to the literature by proposing a new theoretical framework that integrates technological innovation and international trade dynamics to explain divergent paths in employment and value-added shares across sectors. It shows that any surplus profit obtained by domestic monopolistic entrepreneurs due to foreign demand in a specific sector, rather than domestic demand, will widen the disparity between value-added and employment shares. The model illustrates how innovation, through monopoly rights alongside international trade, serves as an extra factor explaining differences between measures of structural change. By addressing gaps in traditional theories, this framework offers a comprehensive understanding of the drivers of structural transformation and provides insights into the dynamics of economic evolution.

The remainder of this paper is structured as follows: Section 2 reviews the various mechanisms

[2] The works of Duarte & Restuccia (2020) , Rodrik (2016), Sposi (2019), Święcki (2017), and Matsuyama (2019) have shed considerable light on the factors influencing structural change as well as the differences between developing and developed countries in the trajectories of structural change in their economies.

through which structural change occurs and how the literature accounts for them. In Section 3, the closed economic model is introduced, which captures the supply-side and demand-side forces driving structural transformation and derives the relationships between different measures of structural change. Section 4 extends the model to incorporate international trade and demonstrates how these relationships are modified by international trade in monopolistic goods. The paper concludes with Section 5, summarizing key insights including some suggested directions for future research.

2 Sources of Structural Change

In this section, we will delve into the mechanisms driving structural change within economies. The two primary mechanisms of structural change are income effects and relative price effects. Income effects, also known as non-homothetic Engel curves, are related to the demand side. On the other hand, relative price effects, also known as Baumol effects, are related to the supply side. These two mechanisms will give rise to additional factors such as international trade, intermediate goods and input-output relations, labor "wedges," investment rates, and trade imbalances.

2.1 Non-Homothetic Preferences: Engel's Law

The Engel's Law, which suggests that as countries become wealthier, they tend to shift their preferences from agriculture-related products to manufacturing and services. This is because the income elasticity of demand for different types of goods is not uniform, and preferences change with increasing wealth. The Engel's Law mechanism is captured by structural change models that define non-homothetic preferences over sectoral goods. Stone-Geary preferences are the most commonly used example of such preferences :

$$U(C_a, C_m, C_s) = \left(C_a - \bar{C}_a\right)^{\omega_a} C_m^{\omega_m} C_s^{\omega_s} \tag{2.1}$$

The Stone-Geary preferences model captures the non-homothetic nature of preferences over sectoral goods, with C_k, $k = a, m, s$ representing consumption in sector k and $\bar{C}_a > 0$ representing the subsistence level of consumption of agriculture-related goods, and where ω_k, $k = a, m, s$ is a weight parameter on consumption. As a country's per-capita income rises, its demand for agriculture-related goods increases at a slower rate than its demand for manufactured goods and services, which increase at a higher rate than one-for-one. As a consequence of the shift in demand away from agriculture-related goods, the share of employment and value-added in agriculture will decrease while the share in manufacturing and services will increase. Since the decline in agriculture is a prominent aspect of structural change, this mechanism is typically incorporated in all structural change models.

However, a limitation of Stone-Geary preferences is that as a country progresses economically, the relevance of the subsistence level of consumption diminishes. In the extreme case, the income elasticity of demand for agriculture-related goods approaches one. Consequently, this has prompted the development of more advanced methods for modeling non-homotheticities such as Constant Elasticity of Substitution (CES) non-homothetic preferences proposed by Comin et al. (2021).

2.2 Asymmetric Sectoral Productivity Growth: Baumol's Law

The second mechanism is Baumol's theory, which argues that goods are not perfect substitutes for each other in consumption. As a result, asymmetric productivity growth in different sectors can lead to structural change. A sector with the highest productivity growth will experience a decline in its relative price because increased productivity means more output can be produced with the same amount of inputs, which will lead to a fall in the price of the goods produced. As the relative price of the goods produced in that sector falls, consumers will shift their demand to other sectors, leading to a decline in expenditure, employment, and value-added shares in the sector with the highest productivity growth. Authors use CES preferences with a non-unitary elasticity of substitution $\sigma \neq 1$ to capture Baumol effect:

$$U(C_a, C_m, C_s) = \left[\omega_a \left(C_a - \bar{C}_a \right)^{\frac{\sigma-1}{\sigma}} + \omega_m C_m^{\frac{\sigma-1}{\sigma}} + \omega_s C_s^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2.2}$$

If $\sigma < 1$, the equilibrium will result in relative prices being inversely proportional to relative productivity, causing prices to decline in sectors with higher productivity growth such as agriculture.

2.3 Additional Mechanisms Driving Structural Change

Expanding upon the aforementioned primary mechanisms, additional factors such as international trade, intermediate goods and input-output relations, labor "wedges," investment rates, and trade imbalances exert influence on structural change primarily through the previously delineated mechanisms.

International trade. There are at least four ways in which international trade impacts structural change. The first is through direct consequences of forces that lead to an increase or decrease in international trade. The second, third, and fourth channels involve the interaction of international trade with the two mechanisms that were discussed earlier. The first channel of how international trade affects structural change is through the direct consequence of forces that lead to an increase or decrease in international trade. For example, if a country has a comparative advantage in manufacturing, the lower costs of international trade will lead to an increase in demand for the country's manufactured goods, resulting in increased employment and value-added in the manufacturing sector. On the other hand, sectors that do not have a comparative advantage will experience decreases in employment and value-added. This reallocation of employment and value-added across sectors due to changing specialization patterns induced by changing trade costs is the first channel.

Increased international trade has a second channel of impact, whereby real income rises, leading to a shift in the demand share from low-income elasticity goods to high income elasticity goods due to non-homothetic preferences. This shift in expenditure will result in changes in employment and value-added sectoral shares. Thus, trade can strengthen the non-homothetic preference mechanism. The third channel is related to the response of relative prices and complementarity in preferences, which is the second mechanism discussed earlier. When trade barriers decline globally, the rates of decline can vary across sectors. For example, industry may experience faster declines than services. This can cause a decline in the relative prices of goods compared to services, leading to a higher expenditure share on services. This, in turn, can result in higher employment shares in the service sector. The fourth channel pertains to the Ricardian mechanism, in which asymmetric sectoral productivity growth between countries affects comparative advantage. As productivity differentials change over time, countries face varying opportunity costs of production, leading to different specialization patterns.

Other mechanisms. In the absence of intermediate goods, there would be a direct, proportional link from final demand to value-added and employment in the global economy or in a closed economy. However, in reality, the share of intermediate goods in production and international trade is high and continues to grow across many sectors. Integrating such crucial aspects of the data into a structural change framework introduces additional channel for structural change. Indeed, changes in sectoral final demand no longer translate directly into changes in sectoral employment or value-added. Instead, these changes propagate across different sectors depending on the degree of cross-sector input-output linkages. For instance, if agriculture goods heavily rely on services, then an increase in demand for food will result in at least a partial increase in demand for services. As demonstrated by Sposi (2019), countries vary in the nature of their input-output linkages based on their level of economic development.

Another additional channel for structural change is policy distortion or other frictions that hinder the equalization of marginal products of labor across sectors. The sectoral reallocation of labor is fundamental to structural change, and most models of structural change assume freely mobile labor. In essence, a higher wedge, such as policy distortion or friction, implies a lower allocation of labor relative to the value added generated in a sector, all else being equal. Furthermore, another channel influencing structural change is investment rates, primarily through final demand. When investment is predominantly comprised of industry (manufacturing and construction), changes in aggregate investment rates will stimulate greater demand for industrial production and employment. A noteworthy trend in the data is the increasing share of investment allocated to services. Consequently, even with fixed aggregate investment rates, this trend could contribute to the growth in services' value added and employment. Finally, trade imbalances, which mean that a country's spending does not equal the value-added of its production. This means there is some "slippage" between changes in final demand and changes in the sectoral allocation of labor. For example, changes in domestic final demand could be met entirely with foreign factors of production with zero effect on domestic labor allocations.

3 The Closed Economy Model

The theoretical foundation of the trade model is provided by the Schumpeterian framework developed by Aghion & Howitt (2009). Additionally, the setup draws upon the work of Comin et al. (2021), who introduced long-run Engel curves to explain the demand-side aspect of structural change. The model incorporates heterogeneous technological innovation to capture the supply-side dynamics of structural change. There are three final sectors - agriculture, manufacturing, and services - indexed by $k = a, m, s$. Each final sector competitively produces a single consumption good, also indexed by $k = a, m, s$, utilizing labor and a continuum of specific intermediate inputs. Time is discrete, indexed by $t = 1, 2, \ldots$, and at each time period, there is a mass L_{jt} of individuals in country j. Each household is endowed with labor units that are supplied inelastically. Sectoral productivity growth arises from innovation within each country.

3.1 Goods production sectors

Final goods production. Each final good k, which is consumed by households, is produced competitively using labor and a unit interval of specific intermediate varieties ν as inputs, according to the Cobb-Douglas production function:

$$Y_{jkt} = L_{jkt}^{1-\alpha} \int_0^1 A_{jkt}(\nu)^{1-\alpha} x_{jkt}(\nu)^{\alpha} \, d\nu \tag{3.1}$$

where $0 < \alpha < 1$ and $A_{jkt}(\nu)$ represents the productivity of the variety ν used in sector k in country j. $x_{jkt}(\nu)$ denotes the input of the latest version of the variety ν used in the production of final good k at time t. L_{jkt} represents the number of workers in country j employed in the production of final good k at time t. Thus, the production function can be expressed as:

$$\sum_{k=a,m,s} L_{jkt} = L_{jt} \quad \forall j \in J \tag{3.2}$$

Since the final sector k operates competitively, the representative firm takes the prices of its output P_{jkt} and inputs $p_{jkt}(\nu)$ as given. It then chooses the quantity of labor L_{jkt} and the quantity $x_{jk}(\nu)$ of each intermediate good ν to use in order to maximize its profit, as follows:

$$\max_{\left\{L_{jkt},[x_{jkt}(\nu)]_{\nu\in[0,1]}\right\}} P_{jkt}L_{jkt}^{1-\alpha}\int_0^1 A_{jkt}(\nu)^{1-\alpha}x_{jkt}(\nu)^{\alpha}\,d\nu - \int_0^1 p_{jkt}(\nu)x_{jkt}(\nu)\,d\nu - w_{jt}L_{jkt} \tag{3.3}$$

where w_{jt} is the wage rate in the country j at period t.

Intermediate goods production. Each variety ν of the final good k is produced by a patent monopoly obtained through an innovation. The lifetime of the patent lasts for a one period. The production technology of intermediate varieties involves using one unit of the final good k to produce a unit of an intermediate variety ν for the sector k. In addition, in every intermediate sector, there are an unlimited number of people capable of producing copies of the latest version of that intermediate variety ν at a unit cost of $\chi_{jkt} > P_{jkt}$[3]. In each period, one entrepreneur succeeds in innovation in a sector and is able to produce at a lower cost than others. Innovations are assumed to be drastic; that the intermediate monopolist is unconstrained by potential competition from the previous patent. Then the producer of the variety ν for the sector k in the country j maximizes its profit as follows:

$$\max_{\left\{\mathbf{x}_{jkt}(\nu)\right\}} \pi_{jkt}(\nu) = p_{jkt}(\nu)\mathbf{x}_{jkt}(\nu) - P_{jkt}\mathbf{x}_{jkt}(\nu) \tag{3.4}$$

$$\text{s.t.} \qquad p_{jkt}(\nu) = f^{-1}\left[\mathbf{x}_{jkt}(\nu)\right]$$

where f is the demand function of the final good's producer for the intermediate good ν.

3.2 Innovation and productivity growth

Productivity growth arises from innovations. In each intermediate variety ν of each sector k, in each period, there exists a unique entrepreneur in country j with the potential to innovate in that variety. This entrepreneur acts as the incumbent monopolist, and an innovation would enable them to produce with a productivity or quality parameter $A_{jkt}(\nu) = \gamma_{jk}A_{jkt-1}(\nu)$, where $\gamma_{jk} > 1$. Otherwise, their productivity parameter remains unchanged $A_{jkt}(\nu) = A_{jkt-1}(\nu)$. Let $\mu_{jkt}(\nu)$ denote the probability that innovation occurs in the intermediate sector ν, then

$$A_{jkt+1}(\nu) = \begin{cases} \gamma_{jk}A_{jkt}(\nu) & \text{with probability } \mu_{jk}(\nu) \\ A_{jkt}(\nu) & \text{with probability } 1-\mu_{jk}(\nu) \end{cases}$$

The probability function of innovation is given by the equation (3.5) below:

$$\lambda_j \frac{P_{jk}Z_{jk}(\nu)}{\gamma_{jk}A_{jk}(\nu)} = F\left[\mu_{jk}(\nu)\right], \quad F' > 0,\ F'' > 0,\ F(0) = 0 \tag{3.5}$$

[3] $\chi_{jkt} > P_{jkt}$ implies that the competitive fringe will produce the intermediate good at a higher cost than the innovator. The parameter χ_{jkt} captures technological factors as well as government regulation affecting entry. A higher χ_{jkt} corresponds to a less competitive market.

where λ_j is a parameter representing the extent to which national policies or institutions encourage innovation, and $P_{jkt}Z_{jkt}(\nu)$ denotes the total amount invested in the intermediate sector ν of sector k in country j for research and development (R&D). The innovation cost $P_{jkt}Z_{jkt}(\nu)$ is divided by $\gamma_{jk}A_{jkt}(\nu)$, the targeted productivity parameter, to account for the higher cost of catching up with the most advanced technologies. At equilibrium, an innovator in country j chooses $Z_{jkt}(\nu)$ to maximize the difference between the expected profit $\beta\mu_{jkt}(\nu)\pi_{jkt+1}(\nu)$ and the cost of innovation $P_{jkt}Z_{jkt}(\nu)$, where β is the time discount factor.

3.3 Household

A representative household in each country j supplies inelastically L_{jt} units of labor, which is perfectly mobile across the three final sectors, at the wage rate w_{jt}. The household decides on consumption over time and also on final demand allocations across the three sectors. The lifetime utility of the representative household is defined over a discounted period utility, which is the logarithm of aggregate consumption per capita. Following Comin et al. (2021), the aggregate consumption C_{jt}, in each period, is defined as a generalized, non-homothetic, CES aggregate over the three sector composite goods C_{jkt}, $k = a,m,s$. The real aggregate consumption C_{jt} is described by an implicit function defined as follows:

$$\sum_{k=a,m,s} \delta_k^{1/\sigma} \left(\frac{C_{jkt}}{C_{jt}^{\varepsilon_k}}\right)^{\frac{\sigma-1}{\sigma}} = 1 \tag{3.6}$$

where δ_k are constant weights for each sector in the economy[4], σ is the elasticity of substitution between goods. $\sigma < 1$ such that agricultural and manufacturing goods and services are complements. ε_k define the relative Engel curve for each sectoral output k, representing the income elasticity of demand of sector k. C_{jt} is a nonhomothetic unobservable index of real consumption in country j at time t.

The key insight of equation (3.6) is the parameter ε_k, which governs the degree of nonhomotheticity. This parameter alone differentiates the role of income across sectors. The sector with a greater ε_k is considered a luxury good, which expands in expenditure shares as income rises, all else equal. Comin et al. (2021) show that this specification of nonhomothetic preferences has attractive properties for studying long-run structural change. Unlike Stone-Geary preferences, the elasticity of relative demand does not approach zero as income or consumption rises, as shown in the data. This feature is particularly relevant for the service sector, whose consumption grows more than proportionally, especially at later stages of development[5]. Note that if $\varepsilon_k = 1$, $\forall k$, then equation (3.6) yields:

$$C_{jt} = \left(\sum_{k\in\{a,m,s\}} \delta_k^{1/\sigma} C_{jkt}^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad \text{if} \quad \varepsilon_k = 1 \quad \forall k = a,m,s \tag{3.7}$$

Equation (3.7) represents the composite good when preferences are homothetic, and σ is the within-period elasticity of substitution between consumption categories. Homothetic preferences are therefore a special case where all ε_k are equal to 1.

[4] $\sum_{k=a,m,s} \delta_k = 1$

[5] The rise of the service sector occurs at later stages of development, and to understand this fact, it is necessary that the income elasticity for services does not level off. With Stone-Geary preferences, the home production parameters play an important role only at early stages, but their effect vanishes in the long run (See Buera & Kaboski (2009)).

In each period, the representative household maximizes its utility, in each period by choosing sectoral consumption levels, C_{jkt}, as follow: Given the nonhomothetic CES aggregator (3.6), the intra-temporal household's problem in country j is as follows:

$$\max_{\{C_{jat}, C_{jmt}, C_{jst}\}} \ln C_{jt} \tag{3.8}$$
$$\text{s.t.} \quad \sum_{k\in\{a,m,s\}} P_{jkt}\left(C_{jkt}+Z_{jkt}\right) \leq w_{jt}L_{jt} + \sum_{k=a,m,s} \Pi_{jkt}$$

where $\Pi_{jkt} := \int_0^1 \pi_{jkt}(\nu)d\nu$ is the total profit made in the sector k of the country j. This utility maximization problem (3.8) is equivalent to total expenditure on consumption in agriculture, manufacturing and services minimization problem subject to the implicit CES nonhomothetic aggregator.

3.4 Equilibrium

Definition 1. *For each country j, the timing of the model can be summarized as follows:*

- ***Step 0** : Period t starts with productivities, $A_{jkt}(\nu), \forall \nu \in [0,1]$, inherited from the production and innovation activities of the previous periods;*
- ***Step 1** : The production of intermediate goods then that of final goods takes place;*
- ***Step 2** Innovators choose the optimal amount $Z_{jkt}(\nu)$ to invest in R&D in each intermediate sector $\nu \in [0,1], k = a,m,s$ for the next period.*
- ***Step 3** : Households choose the levels of consumption of goods a,m and s.*

Let's now define and then characterize the competitive equilibrium of the model.

Definition 2. *A competitive equilibrium is :*
– collections of wage rate and prices of final and intermediate goods $\mathbf{p}_j = \left\{w_{jt}, P_{jkt}\right\}_{t=0\,;k=a,m,s}^{\infty} \quad \forall j.$
– consumption allocation decissions $\mathbf{c}_j = \left\{C_{jat}, C_{jmt}, C_{jst}\right\}_{t=0}^{\infty}$ *for the household for all j;*
– labor and intermediate inputs allocation decisions $\mathbf{f}_j = \left\{L_{jkt}, \{x_{jkt}(\nu)\}_{\nu\in[0,1]}\right\}_{t=0\,;k=a,m,s}^{\infty}$ *for firms in final sectors for all j;*
– collection of decisions $\mathbf{i}_j = \left\{Z_{jkt}(\nu), \mathbf{x}_{jkt}(\nu)\right\}_{t=0\,;\nu\in[0,1],k=a,m,s}^{\infty}$ *for producers of intermediate varieties j_k such that:*

- *Given $\mathbf{p}_j$, households solve the problem (3.23) $\forall j$;*
- *Given $\mathbf{p}_j$, final sectors producers solve the problem (3.3) $\forall j$;*
- *Given $\mathbf{p}_j$, varieties' producers maximize its problem;*

And the following markets clearing conditions are verified:

(a) Labour market : $L_{jat} + L_{jmt} + L_{jst} = L_{jt}$ *for all t and j;*

(b) Intermediate varieties markets : $x_{jkt}(\nu) = \mathbf{x}_{jkt}(\nu) \quad \forall \nu \in [0,1]\,;\, \forall k \in \{a,m,s\} \ \forall t \text{ and } \forall j;$

(c) Final goods markets: $Y_{jkt} = C_{jkt} + X_{jkt} + Z_{jkt} \quad \forall k = a,m,s$ *, for each j and for each period.*

where $X_{jkt} := \int_0^1 x_{jkt}(\nu)d\nu$ *and* $Z_{jkt} := \int_0^1 Z_{jkt}(\nu)d\nu$ *are respectively the aggregate production of intermediate varieties and total investment in R&D in sector k.*

3.5 Firms' optimization

The first order conditions for the firm in the final sector k of the country j are given by:

$$\begin{cases} p_{jkt}(\nu) = \alpha P_{jkt} x_{jkt}(\nu)^{\alpha-1} A_{jkt}(\nu)^{1-\alpha} L_{jkt}^{1-\alpha} \quad \forall \nu \in [0,1] \\ w_{jt} = (1-\alpha) P_{jkt} L_{jkt}^{-\alpha} \int_0^1 A_{jkt}(\nu)^{1-\alpha} x_{jkt}(\nu)^{\alpha} \, d\nu \end{cases}$$

Thus, the firm of the final sector equalizes the marginal productivity of labor to the real wage and the demand function for intermediate goods of variety ν for the firm in the final sector is given by :

$$x_{jkt}(\nu) = \alpha^{\frac{1}{1-\alpha}} \left(\frac{p_{jkt}(\nu)}{P_{jkt}} \right)^{-\frac{1}{1-\alpha}} A_{jkt}(\nu) L_{jkt} \quad \forall \nu \in [0,1] \text{ and } k = a,m,s \tag{3.9}$$

By utilizing the demand function from equation (3.9) in problem (3.4), the equilibrium quantity of the variety ν of sector k in country j is given by:

$$x_{jkt}(\nu) = \alpha^{\frac{2}{1-\alpha}} A_{jkt}(\nu) L_{jkt} \tag{3.10}$$

at the price $p_{jkt}(\nu)$ given by :

$$p_{jkt}(\nu) = \alpha^{-1} P_{jkt} \tag{3.11}$$

The profit made by the intermediate monopoly producing the varietyν in sector k is therefore given at equilibrium by:

$$\pi_{jkt}(\nu) = \pi P_{jkt} A_{jkt}(\nu) L_{jkt} \tag{3.12}$$

where $\pi := (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}$. Therefore, the profits generated by each intermediate monopoly depend positively on productivity, the labor force, and the price of the final good in this sector. Indeed, an increase in the output price in a sector positively affects the prices of intermediate goods used in this sector. Additionally, the increase in labor demand in a sector will have the effect of increasing output and, therefore, increasing the demand for varieties that are used in the same Cobb-Douglas production function.

Using the equation (3.12), the innovator maximizes its expected net payoff of the next period, given by:

$$\max_{\{\mu_{jkt}(\nu)\}} \left[\beta \pi \mu_{jkt}(\nu) P_{jkt+1} L_{jkt+1} - \lambda_j^{-1} P_{jkt} \, F\left(\mu_{jkt}(\nu)\right) \right] \gamma_{jk} A_{jkt}(\nu) \tag{3.13}$$

Solving the problem (3.13) yields the same probability of innovation in the same sector $\mu_{jkt}(\nu) = \mu_{jkt}$ with μ_{jkt} given by:

$$\mu_{jkt} = F'^{-1} \left[\beta \pi \lambda_j \frac{P_{jkt+1}}{P_{jkt}} L_{jkt+1} \right] \quad \forall \nu \in [0,1] \text{ and } \forall k = a,m,s. \tag{3.14}$$

In the special case where the research-productivity function F takes the simple quadratic form:

$$F(\mu_{jkt}(\nu)) = \frac{1}{2} \mu_{jkt}(\nu)^2$$

then the innovation probability in the sector k is given by :

$$\mu_{jkt} = \beta \pi \lambda_j \frac{P_{jkt+1}}{P_{jkt}} L_{jkt+1} \quad \forall k = a, m, s. \tag{3.15}$$

The equation (3.15) indicates that an increase in the demand for labor and in the output price growth rate in a sector encourages entrepreneurs to innovate more in that sector, as the expected gains will increase. All else being equal, it is more profitable to innovate in a larger sector because a successful innovator has a larger market share in that sector. Additionally, considering any changes in demand composition due to Engel's law that increase the demand for sector k, its price will increase, thereby enhancing innovation opportunities due to the higher profitability associated with this sector.

3.6 Aggregate behavior

Let's define the productivity of the sector k in the country j A_{jkt} at time t as :

$$A_{jkt} := \int_0^1 A_{jkt}(\nu)\, d\nu \tag{3.16}$$

Then, the expected productivity growth rate $g_{A_{jkt}}$ of the sector k is determined by :

$$g_{A_{jkt}} = \mathbb{E}_t \left[\frac{A_{jkt+1} - A_{jkt}}{A_{jkt}} \right]$$

Using the expression of μ_{jkt} from equation (3.15) and after a few manipulations[6], the productivity growth rate $g_{A_{kt}}$ is found as the following function:

$$g_{A_{jkt}} = \beta \pi \lambda_j \frac{P_{jkt+1}}{P_{jkt}} (\gamma_{jk} - 1) L_{jkt+1} \tag{3.17}$$

The productivity $g_{A_{jkt}}$ increases with the sectoral labor share and the output price growth rate. An increase in the number of workers in a sector leads to an augmentation in the production of the final good and the demand for intermediate goods, thereby resulting in increased profits for monopolists operating within that sector. This, in turn, incentivizes further innovation, consequently fostering heightened productivity within the sector. Additionally, a rise in prices within a sector amplifies relative profits for monopolists therein, thereby prompting entrepreneurial endeavors towards innovation within said sector, subsequently yielding a positive impact on its productivity. However, in accordance with the Baumol effect, a high rate of productivity growth within a sector leads to a relative decrease in the price of the final good within that sector.

Changes in demand composition across sectors due to differences in income elasticity, known as Engel's Law, will affect variations in innovation rates and productivity changes across sectors. Therefore, there is an absence of dichotomy between Engel's Law from the demand side and sectoral productivity growth effect from the supply side, as the relative productivity changes across sectors respond endogenously to changes in the relative market sizes caused by economic growth due to Engel's Law, as explained by Matsuyama (2019).

The production level of the final good k in country j at equilibrium is obtained by substituting equation (3.10) into equation (3.1):

$$Y_{jkt} = \alpha^{\frac{2\alpha}{1-\alpha}} A_{jkt} L_{jkt} \tag{3.18}$$

[6] More details are provided in Appendix A.3

and the wage rate is determined from the first-order conditions of the firm in the final sector k by:

$$w_{jt} = \omega P_{jkt} A_{jkt} \tag{3.19}$$

where $\omega := (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}$. As the wage rate is constant across sectors in the same country, a slower productivity growth in a sector causes its relative price to go up over time. To see this, let's divide the equation (3.19) for the sector k and m for example, then we can deduce a relation between the price and the productivity in sector k relative to the sector m as shown below:

$$\frac{P_{kt}}{P_{mt}} = \frac{A_{mt}}{A_{kt}} \tag{3.20}$$

Let's denote VA_{jkt} the value added of the sector k and its intermediate branches at the period t in the country j. Then[7]

$$\begin{aligned} VA_{jkt} &= P_{jkt} Y_{jkt} - P_{jkt} \int_0^1 x_{jkt}(\nu)\, d\nu \\ &= \zeta P_{jkt} A_{jkt} L_{jkt} \end{aligned} \tag{3.21}$$

where $\zeta := (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}$. As the wage rate w_{jt} is constant across sectors in the same country, the Gross Domestic Production of the economy is given by :

$$GDP_t = \zeta P_{kt} A_{kt} L_t\,, \quad k = a, m, s \tag{3.22}$$

Note that the GDP is proportional to the nominal wage of the economy and that the sectoral values added are a function of the wage rate and the level of sectoral employment.

3.7 Household's optimization

Given the nonhomothetic CES aggregator, the intra-temporal household's problem in country j is equivalent[8] to:

$$\min_{\{C_{jat}, C_{jmt}, C_{jst}\}} \sum_{k=a,m,s} P_{jkt} C_{jkt} \tag{3.23}$$

$$\text{s.t.} \quad \sum_{k=a,m,s} \delta_k^{1/\sigma} \left(\frac{C_{jkt}}{C_{jt}^{\varepsilon_k}} \right)^{\frac{\sigma-1}{\sigma}} = 1$$

Each period the household minimizes the expenditure on consumption in agriculture, manufacturing and services subject to the implicit CES nonhomothetic aggregator.

The first order conditions[9] imply that sectoral consumption demand satisfies:

$$C_{jkt} = \delta_k \left(\frac{P_{jkt}}{E_{jt}} \right)^{-\sigma} C_{jt}^{\varepsilon_k(1-\sigma)} \tag{3.24}$$

[7] See Appendix A.2 for more details.

[8] The expenditure minimization problem is the dual of the utility maximization problem. The relationship between the utility function and Marshallian demand in the utility maximization problem mirrors the relationship between the expenditure function and Hicksian demand in the expenditure minimization problem.

[9] See Appendix A.4 for calculation.

where $E_{jt} := \sum_{k=a,m,s} P_{jkt}C_{jkt}$ is the total expenditure in consumption at time t in country j. Replacing E_{jt} by $P_{jt}C_{jt}$ in the equation (3.24) where P_{jt} is the average cost of real consumption, one can show that :

$$C_{jkt} = \delta_k \left(\frac{P_{jkt}}{P_{jt}} \right)^{-\sigma} C_{jt}^{\varepsilon_k(1-\sigma)+\sigma} \tag{3.25}$$

where the aggregate price P_{jt} is given[10] by :

$$P_{jt} = \left[\sum_{k=a,m,s} \delta_k P_{jkt}^{1-\sigma} C_{jt}^{(1-\sigma)(\varepsilon_k - 1)} \right]^{\frac{1}{1-\sigma}} \tag{3.26}$$

Thus, the sectoral expenditure e_{jkt} in the good k of the country j is given by:

$$\begin{aligned} e_{jkt} &= \frac{P_{jkt}C_{jkt}}{P_{jt}C_{jt}} \\ &= \delta_k \left(\frac{P_{jkt}}{P_{jt}} \right)^{1-\sigma} C_{jt}^{(1-\sigma)(\varepsilon_k - 1)} \qquad \forall k \end{aligned} \tag{3.27}$$

By dividing e_{jkt} by e_{jmt} using (3.27) and the equation (3.20), we can obtain the expression of the consumption expenditure share of sector $k = a, s$ relative to manufacturing sector m below:

$$\frac{e_{jkt}}{e_{jmt}} = \frac{\delta_k}{\delta_m} \left(\frac{A_{jmt}}{A_{jkt}} \right)^{1-\sigma} C_{jt}^{(\varepsilon_k - \varepsilon_m)(1-\sigma)} \qquad k = a, s \tag{3.28}$$

The equation (3.28) illustrates both the supply and demand-side mechanisms for structural change through the allocation of consumption between different sectors. The parameter σ governs the supply-side mechanisms of the structural change via sector-biased productivity effects, and the relative comparison of income elasticities $\varepsilon_k - \varepsilon_m$ governs the relative long-run Engel curves.

As $\sigma < 1$ due to the complementary nature of foods, manufacturing goods, and services, an increase in the relative sectoral productivity of sector k will result in a decrease in its relative consumption expenditure share. This rise in sector k's productivity will, in turn, lead to a reduction in its final good price. Consequently, consumers can maintain the same quantity of goods from sector k' experiencing greater productivity growth while spending less, enabling them to allocate the remaining income to other products. This ensures a certain level of consumption from sectors with lower productivity growth, ultimately resulting in a decrease in the proportion of expenditure in sectors with higher rates of productivity growth.

Similarly, when sectoral income elasticities differ, such that $\varepsilon_k - \varepsilon_m > 0$, then sector k expenditure share also rises with the aggregate consumption and vice versa. In fact, income elasticity measures how sensitive the demand for a good is to changes in income. When $\varepsilon_k - \varepsilon_m > 0$, it means that the income elasticity of sector k is greater than that of sector m. This implies that as aggregate consumption increases - due to an increase in income-, consumers tend to spend a larger proportion of their income on goods from sector k compared to sector m. Conversely, when $\varepsilon_k - \varepsilon_m < 0$, the expenditure share on goods from sector k decreases relative to sector m. This illustrates how alterations in aggregate consumption influence the distribution of expenditure among sectors with varying income elasticities. In the data, $\varepsilon_a < \varepsilon_m < \varepsilon_s$, indicating that the shares of consumption expenditure on services increase relative to manufacturing, while those on agriculture decrease.

[10] See Appendix A.4 for the demonstration.

3.8 Innovation and Structural Change in a Closed Economy

Structural change is defined as a shift in the relative importance of the aggregate indicators of the economy, such as sectoral national product s_{jkt}, sectoral consumption expenditure e_{jkt}, and the sectoral employment $\ell_{jkt} := \frac{L_{jkt}}{L_{jt}}$ for $k = a, m, s$. By leveraging equations (3.21) and (3.22), I derive the equivalence

$$\ell_{jkt} = s_{jkt} \tag{3.29}$$

Equation (3.29) implies that the share of value added is identical to the share of employment in a closed economic system. This relationship arises from the fact that profits accrued within a particular sector correlate directly with the magnitude of employment within that sector. This association stems from the fact that as the proportion of employment grows within a sector, there is a concurrent increase in the output and in the demand for intermediate goods. Hence, the income of both workers and entrepreneurs follows a linear pattern determined by the wage rate and the employment level within the sector utilizing intermediate goods. Likewise, the market clearing condition for the final good of the sector k is given by:

$$Y_{jkt} = C_{jkt} + X_{jkt} + Z_{jkt} \tag{3.30}$$

Then, we can derive the value added $VA_{jkt} := P_{jkt}Y_{jkt} - P_{jkt}X_{jkt}$ of sector k at time t in country j as follows:

$$VA_{jkt} = P_{jkt}C_{jkt} + P_{jkt}Z_{jkt} \tag{3.31}$$

From equation (3.31), we can express the consumption expenditure share in sector k as follows:

$$e_{jkt} = \frac{VA_{jkt} - P_{jkt}Z_{jkt}}{GDP_{jt} - \sum_{k=a,m,s} P_{jkt}Z_{jkt}} \tag{3.32}$$

Then, by dividing both the numerator and the denominator by GDP and rearranging, we obtain the following equation, which provides the relationship between the share of consumption expenditure, value added, the sectoral share of innovation expenditure, and the the weight of innovation expenditure in GDP:

$$s_{jkt} = \frac{P_{jkt}Z_{jkt}}{GDP_{jt}} + \left(1 - \frac{\sum_{k=a,m,s} P_{jkt}Z_{jkt}}{GDP_{jt}}\right) e_{jkt} \tag{3.33}$$

The equation (3.33) illustrates the disparity between the value-added share and the consumption expenditure share, even in a closed economy. In the subsequent section, I extend the model to incorporate international trade in intermediate goods and derive variations in sectoral value-added and employment shares.

4 Opening the Economy

In this section, trade in intermediate goods between the domestic country and the rest of the world is introduced. For simplicity, I consider two countries, denoted by $J = \{H, F\}$, where H represents the home country and F the foreign country. An identical range of intermediate goods is assumed, with transportation costs between countries. The immediate effect of this opening up is to allow each country to benefit from increased productive efficiency. Within each intermediate sector ν of the final sector k, the world market can then be monopolized by the lowest-cost producer of the latest version of the intermediate good ν of the sector k. The difference from the Eaton-Kortum model of international trade is that, following Aghion & Howitt (2009), I assume that the firm in the final sector uses the latest version of the world intermediate good, the price of which is determined by the monopolist. Transportation costs and other barriers to trade are modeled as exogenous iceberg costs. Specifically, if one unit of variety ν of sector k is shipped from country i, then $\frac{1}{\tau_{ijk}}$ units arrive in country j, with[11] $\tau_{jjk} = 1\ ;\forall k = a,m,s$.

4.1 Final Goods Production

The producer of the final good in sector k utilizes domestic labor and a continuum of intermediate goods produced by global monopolists. These intermediate goods possess a quality or efficiency $\hat{A}_{kt}(\nu) :=\ \max\{A_{Hkt}(\nu), A_{Fkt}(\nu)\}$. In other words, the country that manages to produce the most productive latest version of intermediate good ν in sector k holds the monopoly rights and will be the only one able to commercialize the intermediate good desired by the final sector firms. The Schumpeterian paradigm assumes that producers of final goods employ the latest versions of new technologies or machinery. In country j, the problem faced by the producer of the final good k can be formulated as follows:

$$\max_{\left\{L_{jkt}, [x_{jkt}(\nu)]_{\nu\in[0,1]}\right\}} P_{jkt} L_{jkt}^{1-\alpha} \int_0^1 \hat{A}_{kt}(\nu)^{1-\alpha} x_{jkt}(\nu)^{\alpha}\, d\nu - \int_0^1 \hat{p}_{jkt}(\nu) x_{jkt}(\nu)\, d\nu - w_{jt} L_{jkt} \tag{4.1}$$

where the price at which firm in final sector k in country j buys the latest version of its variety ν is $\hat{p}_{jkt}(\nu)$ given by:

$$\hat{p}_{jkt}(\nu) = \begin{cases} p_{jkt}(\nu) & \text{if} \quad \hat{A}_{kt}(\nu) = A_{jkt}(\nu) \\ \tau_{ijk} p_{ikt}(\nu) & \text{if} \quad \hat{A}_{kt}(\nu) > A_{jkt}(\nu) \end{cases}$$

with $p_{jkt}(\nu) = \alpha^{-1} P_{jkt}\ \ \forall j \neq i \in J$. The demands[12] of the final sector firm k in country j for the intermediate varieties produced in the country j, x^j_{jkt}, and the intermediate varieties produced by the country i, x^i_{jkt} are then given by:

$$\begin{cases} x^j_{jkt}(\nu) = \left(\dfrac{p_{jkt}(\nu)}{\alpha P_{jkt}}\right)^{\frac{1}{\alpha-1}} A_{jkt}(\nu) L_{jkt} \\ x^i_{jkt}(\nu) = \left(\dfrac{\tau_{ijk} p_{ikt}(\nu)}{\alpha P_{jkt}}\right)^{\frac{1}{\alpha-1}} A_{ikt}(\nu) L_{jkt} \end{cases}$$

[11] There is no trade costs within a country.

[12] The intermediate variety ν used by the firm producing the final good depends on the country producing the latest version of the variety ν :

$$x_{jkt}(\nu) = \begin{cases} x^j_{jkt}(\nu) & \text{if} \quad \hat{A}_{kt}(\nu) = A_{jkt}(\nu) \\ x^i_{jkt}(\nu) & \text{if} \quad \hat{A}_{kt}(\nu) = A_{ikt}(\nu) \end{cases}$$

4.2 World Monopoly Varieties Producers

The problem faced by a monopoly firm producing $X^j_{kt}(\nu)$ quantity of the variety ν in sector k in country j for the world market is given by:

$$\max_{\{p_{jkt}(\nu)\}} \pi_{jkt}(\nu) = p_{jkt}(\nu)X^j_{kt}(\nu) - P_{jkt}X^j_{kt}(\nu) \tag{4.2}$$

$$\text{s.t.} \quad \begin{cases} X^j_{kt}(\nu) = x^j_{jkt}(\nu) + x^j_{ikt}(\nu) \\ x^j_{jkt}(\nu) = \left(\dfrac{p_{jkt}(\nu)}{\alpha P_{jkt}}\right)^{\frac{1}{\alpha-1}} A_{jkt}(\nu)L_{jkt} \\ x^j_{ikt}(\nu) = \left(\dfrac{\tau_{jik}p_{jkt}(\nu)}{\alpha P_{ikt}}\right)^{\frac{1}{\alpha-1}} A_{jkt}(\nu)L_{ikt} \end{cases}$$

By solving the problem (4.2), as in the case of the closed economy, the monopolist will choose the price level that maximizes its profit, namely, $p_{jkt}(\nu) = \alpha^{-1}P_{jkt}$. Then, the demands of the country j final good k producer are expressed as follows:

$$\begin{cases} x^j_{jkt}(\nu) = \alpha^{\frac{2}{1-\alpha}} A_{jkt}(\nu)L_{jkt} \\ x^i_{jkt}(\nu) = \alpha^{\frac{2}{1-\alpha}} \left(\dfrac{P_{jkt}}{\tau_{ijk}P_{ikt}}\right)^{\frac{1}{1-\alpha}} A_{ikt}(\nu)L_{jkt} \end{cases}$$

Thus, the profit of a world monopoly firm producing the variety ν in the sector k in country j is given by:

$$\begin{aligned} \pi_{jkt}(\nu) &= \left(\alpha^{-1} - 1\right) P_{jkt} \left[x^j_{jkt}(\nu) + x^j_{ikt}(\nu)\right] \\ &= \pi P_{jkt} A_{jkt}(\nu) \left[L_{jkt} + \left(\frac{P_{ikt}}{\tau_{ijk}P_{jkt}}\right)^{\frac{1}{1-\alpha}} L_{ikt}\right] \end{aligned} \tag{4.3}$$

with $\pi := (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}$. The total profit Π_{jkt} made in the sector k in the country j at time t is :

$$\Pi_{jkt} = \pi P_{jkt} \left[L_{jkt} + \left(\frac{P_{ikt}}{\tau_{ijk}P_{jkt}}\right)^{\frac{1}{1-\alpha}} L_{ikt}\right] \int_{\Theta_{jkt}} A_{jkt}(\nu)d\nu \tag{4.4}$$

where Θ_{jkt} is the set of varieties of the sector k that country j produces and exports at time t such that :

$$\int_{\Theta_{jkt}} A_{jkt}(\nu)d\nu = \int_0^1 A_{jkt}(\nu)\mathbb{1}_{\{A_{jkt}(\nu)>A_{ikt}(\nu)\}}d\nu$$

The equation (4.4) illustrates that the profits of domestic monopolists in a given sector k are proportional not only to the domestic employment but also to the foreign employment size. A portion of the profit stems from foreign demand, which correlates with the size of the partner country's sector. Sectors in which the country holds global monopoly experience increased profits compared to a closed economy, whereas sectors in which the country imports all intermediate goods see their profits diminish to zero.

Using the first order conditions of the problem (4.1), the wage rate in the country j, w_{jt}, at time t is determined by :

$$w_{jt} = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} P_{jkt} A_{jkt} \tag{4.5}$$

where the sectoral aggregate productivity of country j in sector k at time t, denoted as A_{jkt}, is determined by:

$$A_{jkt} = \int_{\Theta_{jkt}} A_{jkt}(\nu)d\nu + \left(\frac{P_{jkt}}{\tau_{ijk}P_{ikt}}\right)^{\frac{\alpha}{1-\alpha}} \int_{\Theta_{ikt}} A_{ikt}(\nu)d\nu \tag{4.6}$$

Finally, the equilibrium condition in the final goods market is described by the equation (4.7) below:

$$Y_{jkt} = C_{jkt} + Z_{jkt} + \int_{\Theta_{jkt}} \left[x^j_{jkt}(\nu) + x^j_{ikt}(\nu)\right] d\nu \tag{4.7}$$

The condition (4.7) stipulates that, in each sector-country, the utilization of the final good k must equal its supply. This utilization comprises consumption and investment in R&D by the representative household, as well as the use of intermediate inputs by innovator firms producing the varieties for both domestic and foreign final good k producers.

4.3 Monopoly Rights, Trade, and Structural Change

In this subsection, I will establish new relationships between sectoral shares of value added and shares of consumption expenditure on one hand, and sectoral shares of employment and value added on the other hand. I will then analyze how innovation and monopoly rights through international trade modify the pre-established relationships in a closed economy. Let us begin by first expressing a relationship between sectoral shares of value added and those of consumption expenditure.

Comparison of Consumption Expenditure and Value Added Shares. The value added of the sector k in the country j is defined by :

$$\begin{aligned} VA_{jkt} = & \underbrace{P_{jkt}Y_{jkt} - \int_{\Theta_{jkt}} p_{jkt}(\nu)x^j_{jkt}(\nu)\,d\nu - \int_{\Theta_{ikt}} \tau_{ijk}p_{ikt}(\nu)x^i_{jkt}(\nu)\,d\nu}_{\text{Final sector value added}} \\ & + \underbrace{\int_{\Theta_{jkt}} \left(p_{jkt}(\nu) - P_{jkt}\right)\left[x^j_{jkt}(\nu) + x^j_{ikt}(\nu)\right] d\nu}_{\text{Intermediate varieties value added}} \end{aligned} \tag{4.8}$$

Using the equation (4.7) of the final good market clearing condition, the equation (4.8) transforms into the equation below:

$$VA_{jkt} = P_{jkt}\left(C_{jkt} + Z_{jkt}\right) + NX_{jkt} \tag{4.9}$$

where NX_{jkt} represents the net exports of all varieties of sector k for country j at period t:

$$NX_{jkt} = \int_{\Theta_{jkt}} p_{jkt}(\nu)x^j_{ikt}(\nu)\,d\nu - \int_{\Theta_{ikt}} \tau_{ijk}p_{ikt}(\nu)x^i_{jkt}(\nu)\,d\nu \tag{4.10}$$

Now, from equation (4.9), the consumption expenditure share in sector k is given by:

$$e_{jkt} = \frac{VA_{jkt} - NX_{jkt} - P_{jkt}Z_{jkt}}{GDP_{jt} - \sum_{k=a,m,s} P_{jkt}Z_{jkt} - \sum_{k=a,m,s} NX_{jkt}} \tag{4.11}$$

Dividing both the numerator and the denominator by GDP_{jt} and rearranging, I obtain the relation below :

$$s_{jkt} = \left(1 - \frac{\sum_{k=a,m,s} P_{jkt}Z_{jkt}}{GDP_{jt}} - \frac{\sum_{k=a,m,s} NX_{jkt}}{GDP_{jt}}\right) e_{jkt} + \frac{P_{jkt}Z_{jkt} + NX_{jkt}}{GDP_{jt}} \tag{4.12}$$

where s_{jkt} is the value added share of the sector k in the country j at time t. The equation (4.12) illustrates that net export expenditures, in addition to research and development spending in each sector, alter the relationship between sectoral value added shares and consumption expenditure shares. If a country exports relatively more than it imports in a given sector k, or allocates relatively higher expenditures in this sector, then the share of value added in this sector is likely to be higher than that of consumption expenditure, and vice versa.

Comparison of Employment and Value Added Shares. Let us now establish the relationship between the share of value added and the sectoral labor share in each country. The value added in sector k can also be defined from a revenue perspective as the sum of wages and all profits in sector k as follows:

$$VA_{jkt} = w_{jt}L_{jkt} + \Pi_{jkt} \tag{4.13}$$

Utilizing the expression of Π_{jkt} provided in equation (4.4), the value added share is given by:

$$s_{jkt} = \frac{w_{jt}L_{jkt} + \pi P_{jkt}\left[L_{jkt} + \left(\frac{P_{ikt}}{\tau_{ijk}P_{jkt}}\right)^{\frac{1}{1-\alpha}} L_{ikt}\right]\int_{\Theta_{jkt}} A_{jkt}(\nu)d\nu}{w_{jt}L_{jt} + \sum_{n=a,m,s} \pi P_{jnt}\left[L_{jnt} + \left(\frac{P_{int}}{\tau_{ijn}P_{jnt}}\right)^{\frac{1}{1-\alpha}} L_{int}\right]\int_{\Theta_{jnt}} A_{jnt}(\nu)d\nu} \tag{4.14}$$

Then, by dividing both the numerator and the denominator by $w_{jt}L_{jt}$ and rearranging to make the sectoral employment share $\ell_{jkt} := \frac{L_{jkt}}{L_{jt}}$ appear in each term, we obtain the following relationship between the sectoral value added share s_{jkt} and the sectoral employment share ℓ_{jkt}:

$$\frac{s_{jkt}}{\ell_{jkt}} = \frac{1 + \alpha\left[1 + \left(\frac{P_{ikt}}{\tau_{ijk}P_{jkt}}\right)^{\frac{1}{1-\alpha}} \frac{L_{ikt}}{L_{jkt}}\right]\frac{1}{A_{jkt}}\int_{\Theta_{jkt}} A_{jkt}(\nu)d\nu}{1 + \alpha \sum_{n=a,m,s} \ell_{jnt}\left[1 + \left(\frac{P_{int}}{\tau_{ijn}P_{jnt}}\right)^{\frac{1}{1-\alpha}} \frac{L_{int}}{L_{jnt}}\right]\frac{1}{A_{jnt}}\int_{\Theta_{jnt}} A_{jnt}(\nu)d\nu} \tag{4.15}$$

The relationship between sectoral shares of value added and employment, as described by Equation (4.15), underscores the absence of inherent equality between these two measures *a priori*. Equality between them emerges when total profits across sectors are equal, a condition typically realized when the nation lacks monopolistic control in any sector and imports all intermediate goods. In sectors characterized by high levels of innovation on the global stage, where domestic companies act as monopolists, surplus profits are often generated. This innovation-driven competitiveness enables these firms to command higher prices for their goods or services, resulting in a larger share of value added within the sector. Consequently, while the share of sectoral value added tends to exceed that of employment in such innovative sectors, it tends to be lower in sectors where domestic firms exhibit lower levels of innovation compared to their international counterparts.

Furthermore, it is important to note that the structure of the economy in the rest of the world also determines the relationship between sectoral shares of value added and sectoral shares of employment. If the size of sector k is relatively larger compared to other sectors $n \neq k$ in the foreign country i, such that $\frac{L_{ikt}}{L_{jkt}} > \frac{L_{int}}{L_{jnt}}$ for $n \neq k$, then the share of value added in sector k in the home country j tends to increase relative to the share of employment in sector k. This implies an interconnectedness of national economies with the global economic landscape. Changes in the structure and performance of foreign economies can have significant implications for domestic sectors, affecting their relative shares of value added and employment. These linkages manifest through the demand for intermediate goods, which serves as a crucial channel for transmitting economic changes across borders. When a foreign economy experiences substantial growth or possesses a significant size in particular sectors, it often translates into a higher demand for intermediate goods produced by domestic world monopolists. This heightened demand stems from the need for inputs and components necessary for the production processes within the larger foreign sectors. As a result, domestic producers of intermediate goods experience increased sales and profitability, driving up their share of value added within the economy. Additionally, when the ratio of the price of final goods in the foreign country to the national price is higher in a given sector k, such that $\frac{P_{ikt}}{\tau_{ijk} P_{jkt}} > \frac{P_{int}}{\tau_{ijn} P_{jnt}}$ for $k \neq n$, then the additional profits in sector k are higher, and the share of value added increases more in sector k than it does in sector n. Indeed, the demand for intermediate goods from the foreign country decreases with the cost related to trade and with the price of the exporting country, and it increases with the price of the final goods from the importing country.

5 Conclusion

The analysis presented in this paper underscores the intricacies of economic structural change and its measures, particularly focusing on sectoral employment shares, sectoral value-added shares, and sectoral final consumption expenditure shares. While previous research often treated these measures as interchangeable, Buera & Kaboski (2009) and Herrendorf et al. (2014) highlighted their quantitative distinctions.

This paper proposes a novel theoretical framework grounded in a Schumpeterian paradigm, which integrates technological innovation and international trade dynamics to explain the disparities between sectoral value-added and sectoral employment shares. By considering surplus profits obtained by domestic monopolistic entrepreneurs due to foreign demand alongside domestic demand, this framework offers a comprehensive understanding of the drivers of structural transformation. It sheds light on the factors contributing to disparities between value-added and employment shares across sectors, filling critical gaps in traditional theories.

The model shows that sectors characterized by high levels of innovation and monopolistic control by domestic companies tend to generate surplus profits by charging higher prices for their products, consequently leading to a larger share of value added within their sector compared to employment. Moreover, the structure and performance of foreign economies significantly impact this relationship, as larger or rapidly growing foreign sectors drive a heightened demand for intermediate goods from domestic producers. This increased demand not only boosts sales and profitability but also enhances the share of value added relative to employment. Furthermore, lower trade costs and higher foreign prices stimulate increased demand for intermediate goods, further amplifying the share of value added within sectors.

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A Appendix

A.1 Goods production sectors

The problem of the intermediate firm j_k with drastic innovation is :

$$\max_{\{x_{jkt}(\nu)\}} \pi_{jkt}(\nu) = p_{jkt}(\nu)x_{jkt}(\nu) - P_{jkt}x_{jkt}(\nu) \tag{A.1}$$

$$\text{s.t.} \qquad p_{jkt}(\nu) = \alpha P_{jkt}x_{jkt}(\nu)^{\alpha-1}A_{jkt}(\nu)^{1-\alpha}L_{jkt}^{1-\alpha}$$

The first order condition is given by:

$$\alpha^2 P_{jkt}x_{jkt}(\nu)^{\alpha-1}A_{jkt}(\nu)^{1-\alpha}L_{jkt} - P_{jkt} = 0 \Longleftrightarrow x_{jkt}(\nu) = \alpha^{\frac{2}{1-\alpha}}A_{jkt}(\nu)L_{jkt}$$

Then the intermediate variety price is given from the constraint of the problem (A.1) by :

$$p_{jkt}(\nu) = \alpha^{-1}P_{jkt} \tag{A.2}$$

A.2 Aggregate behavior

The value added of the sector k in the country j in closed economy is given by :

$$\begin{aligned}
VA_{jkt} &= \underbrace{P_{jkt}Y_{jkt} - \int_0^1 p_{jkt}(\nu)x_{jkt}(\nu)\,d\nu}_{\text{Final sector value added}} + \underbrace{\int_0^1 \left(p_{jkt}(\nu)x_{jkt}(\nu) - P_{jkt}x_{jkt}(\nu)\right)d\nu}_{\text{Intermediate varieties value added}} \\
&= P_{jkt}Y_{jkt} - \int_0^1 P_{jkt}x_{jkt}(\nu)\,d\nu \\
&= \alpha^{\frac{2\alpha}{1-\alpha}}P_{jkt}A_{jkt}L_{jkt} - \alpha^{\frac{2}{1-\alpha}}P_{jkt}A_{jkt}L_{jkt} \\
&= (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}P_{jkt}A_{jkt}L_{jkt}
\end{aligned}$$

❖ *Calculation of GDP by income perspective*

$$GDP_{jt} = w_{jt}L_{jt} + \sum_{k=a,m,s}\Pi_{jkt} \tag{A.3}$$

where

$$\Pi_{jkt} = \int_0^1 \pi_{jkt}(\nu)\,d\nu$$

is the total profits made in sector k intermediate varieties. By replacing w_{jt} and $\pi_{jkt}(\nu)$ by their expression, the equation (A.3) becomes :

$$\begin{aligned}
GDP_{jt} &= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}P_{jkt}A_{jkt}L_{jkt} + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\sum_{n=a,m,s}P_{jnt}A_{jnt}L_{jnt} \\
&= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}P_{jkt}A_{jkt} + (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}P_{jkt}A_{jkt}\sum_{n=a,m,s}L_{jnt} \\
&= (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\left[1+\alpha^{\frac{1+\alpha-2\alpha}{1-\alpha}}\right]P_{jkt}A_{jkt}L_{jt} \\
&= \zeta P_{jkt}A_{jkt}L_{jt} \quad \forall k = a,m,s
\end{aligned} \tag{A.4}$$

where $\zeta := (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}$

❖ *Calculation of the GDP by value added perspective*

$$\begin{aligned} GDP_{jt} &= \sum_{n=a,m,s} VA_{jnt} \\ &= \sum_{n=a,m,s} \zeta P_{jnt} A_{jnt} L_{jnt} \\ &= \zeta P_{jkt} A_{jkt} \sum_{n=a,m,s} L_{jnt} \\ &= \zeta P_{jkt} A_{jkt} L_{jt} \quad \forall k = a,m,s \end{aligned} \tag{A.5}$$

A.3 Dynamics of productivity

The expected productivity growth rate $g_{A_{kt}}$ of the sector k is :

$$\begin{aligned} g_{A_{jkt}} &= \frac{A_{jkt+1} - A_{jkt}}{A_{jkt}} \\ &= \frac{1}{A_{jkt}} \int_0^1 \mu_{jkt} \left(\gamma_{jk} A_{jkt}(\nu) - A_{jkt}(\nu)\right) \, d\nu \\ &= \frac{\mu_{jkt}(\gamma_{jk}-1)}{A_{kt}} \int_0^1 A_{jkt}(\nu) \, d\nu \\ &= \mu_{jkt}(\gamma_{jk}-1) \end{aligned} \tag{A.6}$$

A.4 Housesolds' optimization

The lagragian of the household's problem in country j is :

$$\mathscr{L}\left(C_{jat}, C_{jmt}, C_{jst}; \eta_j\right) = \sum_{k=a,m,s} P_{jkt} C_{jkt} + \eta_j \left[1 - \delta_k^{1/\sigma} \left(\frac{C_{jkt}}{C_{jt}^{\varepsilon_k}}\right)^{\frac{\sigma-1}{\sigma}}\right] \tag{A.7}$$

where η_j is the Lagrange multiplier. The first order conditions are given by :

$$\frac{\partial \mathscr{L}}{\partial C_{jkt}} = P_{jkt} - \eta_j \delta_k^{1/\sigma} \left(\frac{\sigma-1}{\sigma}\right) \frac{C_{jt}^{\varepsilon_k}}{C_{jt}^{2\varepsilon_k}} \left(\frac{C_{jkt}}{C_{jt}^{\varepsilon_k}}\right)^{-1/\sigma} = 0 \quad \forall k = a,m,s \tag{A.8}$$

Then the price of the composite good in the sector k in the country j is given by :

$$P_{jkt} = \eta_j \left(\frac{\sigma-1}{\sigma}\right) \frac{\delta_k^{1/\sigma}}{C_{jt}^{\varepsilon_k}} \left(\frac{C_{jkt}}{C_{jt}^{\varepsilon_k}}\right)^{-\frac{1}{\sigma}} \tag{A.9}$$

And the expenditure in the consumption of the sector k final good is given by :

$$P_{jkt} C_{jkt} = \eta_j \left(\frac{\sigma-1}{\sigma}\right) \delta_k^{1/\sigma} \left(\frac{C_{jkt}}{C_{jt}^{\varepsilon_k}}\right)^{1-\frac{1}{\sigma}} \quad \forall k \tag{A.10}$$

Using the equation (A.10) and the utility function equation (3.6), the total expenditure $E_{jt} := \sum_{k=a,m,s} P_{jkt} C_{jkt}$ in the country j at time t is given by :

$$E_{jt} = \eta_j \left(\frac{\sigma-1}{\sigma}\right) \tag{A.11}$$

The expression (A.9) of the price of the final good of the sector k can be rewriten as :

$$\frac{P_{jkt}}{E_{jt}} = \delta_k^{1/\sigma} C_{jkt}^{-1/\sigma} C_{jt}^{\varepsilon_k\left(\frac{1}{\sigma}-1\right)} \tag{A.12}$$

The the first order conditions imply that :

$$C_{jkt} = \delta_k \left(\frac{P_{kkt}}{E_{jt}}\right)^{-\sigma} C_{jt}^{\varepsilon_k(1-\sigma)} \quad \forall k = a,m,s \tag{A.13}$$

By rasing each of the equations (A.9) to the power $1-\sigma$, then one obtains :

$$P_{jkt}^{1-\sigma} = \delta_k^{\frac{1-\sigma}{\sigma}} E_{jt}^{1-\sigma} C_{jt}^{(\sigma-1)\varepsilon_k} \left(\frac{C_{jkt}}{C_{jt}^{\varepsilon_k}}\right)^{1-\frac{1}{\sigma}} \quad \forall k \tag{A.14}$$

So

$$\delta_k P_{jkt}^{1-\sigma} C_{jt}^{(1-\sigma)\varepsilon_k} = E_{jt}^{1-\sigma} \delta_k^{\frac{1}{\sigma}} \left(\frac{C_{jkt}}{C_{jt}^{\varepsilon_k}}\right)^{1-\frac{1}{\sigma}} \quad \forall k = a,m,s \tag{A.15}$$

By adding the equations (A.15), we obtain :

$$\sum_k \delta_k P_{jkt}^{1-\sigma} C_{jt}^{(1-\sigma)\varepsilon_k} = E_{jt}^{1-\sigma} \tag{A.16}$$

$$\Longrightarrow C_{jt}^{1-\sigma} \sum_k \delta_k P_{jkt}^{1-\sigma} C_{jt}^{(1-\sigma)(\varepsilon_k-1)} = E_{jt}^{1-\sigma} \tag{A.17}$$

By defining the aggregate price P_{jt} in the country j such that $P_{jt}C_{jt} = \sum_k P_{jkt}C_{jkt}$, we can deduce from the equation (A.17) the expression of P_{jt} as follow:

$$P_{jt} = \left[\sum_{n=a,m,s} \delta_n P_{jnt}^{1-\sigma} C_{jt}^{(1-\sigma)(\varepsilon_n-1)}\right]^{\frac{1}{1-\sigma}} \tag{A.18}$$

From the equation (A.13) we can derive the demand for the composite good k in function of the aggregate consumption and aggregate price:

$$C_{jkt} = \delta_k \left(\frac{P_{jkt}}{P_{jt}}\right)^{-\sigma} C_{jt}^{\varepsilon_k(1-\sigma)+\sigma} \quad \forall k = a,m,s \tag{A.19}$$

The expenditure share e_{jkt} of the sector k in the country j is :

$$\begin{aligned} e_{jkt} &= \frac{P_{jkt}C_{jkt}}{P_{jt}C_{jt}} \\ &= \delta_k \left(\frac{P_{jkt}}{P_{jt}}\right)^{1-\sigma} C_{jt}^{(1-\sigma)(\varepsilon_k-1)} \quad \forall k \end{aligned} \tag{A.20}$$

$$\begin{aligned} \Longrightarrow \frac{e_{jkt}}{e_{jmt}} &= \frac{\delta_k}{\delta_m}\left(\frac{P_{jkt}}{P_{jmt}}\right)^{1-\sigma} \times \frac{C_{jt}^{(\varepsilon_k-1)(1-\sigma)}}{C_{jt}^{(\varepsilon_m-1)(1-\sigma)}} \\ &= \frac{\delta_k}{\delta_m}\left(\frac{P_{jkt}}{P_{jmt}}\right)^{1-\sigma} C_{jt}^{(\varepsilon_k-\varepsilon_m)(1-\sigma)} \end{aligned} \tag{A.21}$$

The equation (A.20) gives :

$$e_{jkt} = \delta_k \left(\frac{P_{jkt}}{E_{jt}C_{jt}^{-1}}\right)^{1-\sigma} C_{jt}^{(1-\sigma)(\varepsilon_k-1)}$$
$$= \delta_k \left(\frac{P_{jkt}}{E_{jt}}\right)^{1-\sigma} C_{jt}^{(1-\sigma)\varepsilon_k} \qquad \forall k=a,m,s \tag{A.22}$$

Hence,

$$C_{jt} = \left[\left(\frac{e_{jkt}}{\delta_k}\right)^{\frac{1}{1-\sigma}} \left(\frac{E_{jt}}{P_{jkt}}\right)\right]^{1/\varepsilon_k} \qquad k=a,m,s \tag{A.23}$$

Then the equation (A.21) becomes :

$$\frac{e_{jkt}}{e_{jmt}} = \frac{\delta_k}{\delta_m}\left(\frac{P_{jkt}}{P_{jmt}}\right)^{1-\sigma} \left[\frac{E_{jt}}{P_{jmt}}\left(\frac{e_{jmt}}{\delta_m}\right)^{\frac{1}{1-\sigma}}\right]^{\frac{(1-\sigma)(\varepsilon_k-\varepsilon_m)}{\varepsilon_m}}$$
$$= \frac{\delta_k}{\delta_m}\left(\frac{P_{jkt}}{P_{jmt}}\right)^{1-\sigma} \left(\frac{e_{jmt}}{\delta_m}\right)^{\frac{\varepsilon_k}{\varepsilon_m}-1} \left(\frac{E_{jt}}{P_{jmt}}\right)^{\frac{(1-\sigma)(\varepsilon_k-\varepsilon_m)}{\varepsilon_m}} \tag{A.24}$$

And the expenditure share is finally given by :

$$e_{jkt} = \delta_k \left(\frac{e_{jmt}}{\delta_m}\right)^{\frac{\varepsilon_k}{\varepsilon_m}} \left(\frac{P_{jkt}}{P_{jmt}}\right)^{1-\sigma} \left(\frac{E_{jt}}{P_{jmt}}\right)^{(1-\sigma)\left(\frac{\varepsilon_k}{\varepsilon_m}-1\right)} \tag{A.25}$$