ECN 710 : Advanced Macroeconomics

Chapter 6: Schumpeterian Growth Model

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Introduction

- The Schumpeterian model focuses on growth through creative destruction.
- Key assumptions include the use of labor and vertically differentiated intermediate goods in production.
- The final goods market is perfectly competitive, while the intermediate goods market is monopolistic.
- Innovation depends on resources devoted to R&D, determining the average productivity level.

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Final Goods Production

- The production technology for the final goods sector can be written as:

$$Y_t = L_t^{1-\alpha}\left(\int_0^1 A_t(v)^{1-\alpha} x_t(v)^{\alpha} dv\right)$$

- L_t represents labor demand, $x_t(v)$ the flow of intermediate goods of variety v and quality q, and $A_t(v)$ the highest quality of intermediate goods of variety v.

- The mass of intermediate goods is normalized to 1, so $v \in [0, 1]$.

Profit Maximization

- The representative firm in the final goods sector maximizes its profit:

$$\max_{L_t,[x_t(\nu)]_{\nu\in[0,1]}} \Pi_t = L_t^{1-\alpha}\left(\int_0^1 A_t(\nu)^{1-\alpha} x_t(\nu)^{\alpha} d\nu\right) - \int_0^1 p_t(\nu)x_t(\nu)d\nu - w_t L_t$$

- The firm chooses the quantity of labor and each variety of intermediate goods to use.

- The final goods sector is competitive, so the price of the final good (normalized to 1) and the price of each variety of intermediate goods $p_t(\nu)$ and the real wage w_t are given.

First-Order Conditions

- The first-order conditions for $v \in [0,1]$ are:

$$\frac{\partial \Pi_t}{\partial x_t(v)} = \alpha L_t^{1-\alpha} A_t(v)^{1-\alpha} x_t(v)^{\alpha-1} - p_t(v) = 0$$

$$\frac{\partial \Pi_t}{\partial L_t} = (1-\alpha) L_t^{-\alpha} \left(\int_0^1 A_t(v)^{1-\alpha} x_t(v)^{\alpha} dv \right) - w_t = 0$$

- The firm equates the marginal productivity of each factor to its price.

Inverse Demand Function for Intermediate Goods

- The representative firm producing the final good is willing to pay up to:

$$p_t(v) = \alpha L_t^{1-\alpha} A_t(v)^{1-\alpha} x_t(v)^{\alpha-1}$$

- The inverse demand function for intermediate goods of variety v and quality $A_t(v)$ is:

$$x_t(v) = \left(\frac{\alpha}{1-\alpha}\right) p_t(v)^{-\frac{1}{1-\alpha}} L_t A_t(v)$$

- The price elasticity of demand is constant and given by $\frac{1}{1-\alpha}$ in absolute value.

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Market Structure of the Intermediate Goods Sector

- Each variety v is produced by a monopoly holding a patent obtained through innovation.
- The patent and thus the monopoly last for one period. After one period, a new monopoly appears either through innovation or randomly replacing the old monopoly.
- The intermediate firm makes two choices: (i) quality (innovation) and (ii) quantity produced.
- Innovation is driven by the prospect of profits, so the firm solves for quantity before solving for quality.

Competitive Fringe

- For each variety v, there is an infinite number of firms in a competitive fringe capable of copying the production plans of existing intermediate goods (reverse engineering).

- It takes $\chi > 1$ units of final goods to produce one unit of intermediate goods: $m_t^c(v) = \chi x_t^c(v)$, at a higher cost than the innovator.

- Innovation is drastic if the monopoly sets its monopoly price, and non-drastic if constrained by the less efficient competitive fringe, practicing a limit pricing strategy: $p_t(v) = \chi$.

Profit Maximization of the Intermediate Monopoly

- The intermediate monopoly maximizes its profit:

$$\max_{x_t(v),p_t(v),m_t(v)} \Pi_t(v) = p_t(v)x_t(v) - m_t(v)$$

- Subject to:

$$x_t(v) = m_t(v) \quad \text{(Technological constraint)}$$

$$p_t(v) = \begin{cases} \alpha L_t^{1-\alpha} A_t(v)^{1-\alpha} x_t(v)^{\alpha-1} & \text{(Drastic innovation)} \\ \chi & \text{(Non-drastic innovation)} \end{cases}$$

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Equilibrium Quantity and Price

- For drastic innovation, the equilibrium quantity produced is:

$$x_t(v) = \alpha^{\frac{2}{1-\alpha}} L_t A_t(v)$$

- The equilibrium price is:

$$p_t(v) = \frac{1}{\alpha}$$

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- For non-drastic innovation, the equilibrium quantity produced is:

$$x_t(v) = \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t(v) L_t$$

- The equilibrium price is:

$$p_t(v) = \chi$$

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Equilibrium Profits

The equilibrium profit is :

$$\Pi_t(\nu) = \pi L_t A_t(\nu)$$

- For drastic innovation, :

$$\pi = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}$$

- For non-drastic innovation,

$$\pi = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}$$

Interpretation of Equilibrium Profit

- Equilibrium profit depends positively on the parameter χ, meaning:

  1. **Institutional View:** Higher profits (and thus innovation and growth) are linked to stronger property rights protection, such as patents that increase imitation costs for the competitive fringe.
  2. **Competitive View:** Following the Schumpeterian trade-off, an increase in competition that reduces χ decreases equilibrium profits, lowering innovation incentives. If $\chi = 1$, profits are zero, and there is no incentive to innovate, rendering the model neoclassical.

- **Proof:** The derivative with respect to χ is:

$$\frac{d\pi}{d\chi} = \alpha^{\frac{1}{1-\alpha}} \left[\chi^{-\frac{1}{1-\alpha}} - \chi^{-1}\right] > 0 \text{ since } \chi < \frac{1}{\alpha}. \quad (1)$$

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- The production of equilibrium final goods in the case of non-drastic innovation is :

$$Y_t = L_t^{1-\alpha} \int_0^1 A_t(v)^{1-\alpha} \left[\left(\frac{\alpha}{\chi} \right)^{\frac{1}{1-\alpha}} A_t(v) L_t \right]^{\alpha} dv = \left(\frac{\alpha}{\chi} \right)^{\frac{\alpha}{1-\alpha}} L_t A_t \qquad (2)$$

where:

$$A_t = \int_0^1 A_t(v) dv \qquad (3)$$

represents the average productivity of the economy.

Equilibrium Final Goods Production

- The production of equilibrium final goods in the case of non-drastic innovation is :

$$Y_t = L_t^{1-\alpha} \int_0^1 A_t(v)^{1-\alpha} \left[\left(\frac{\alpha}{\chi} \right)^{\frac{1}{1-\alpha}} A_t(v) L_t \right]^{\alpha} dv = \left(\frac{\alpha}{\chi} \right)^{\frac{\alpha}{1-\alpha}} L_t A_t \tag{2}$$

where:

$$A_t = \int_0^1 A_t(v) dv \tag{3}$$

represents the average productivity of the economy.

- In the case of drastic innovation, the equilibrium final goods production is given by:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_t(v)^{1-\alpha} \left[\alpha^{\frac{1}{1-\alpha}} A_t(v) L_t \right]^{\alpha} dv = \alpha^{\frac{2\alpha}{1-\alpha}} L_t A_t \tag{4}$$

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Real Equilibrium Wages

- The real equilibrium wage in the case of non-drastic innovation is :

$$w_t = (1-\alpha)L_t^{-\alpha} \int_0^1 A_t(v)^{1-\alpha} \left[\left(\frac{\alpha}{\chi} \right)^{\frac{1}{1-\alpha}} A_t(v) L_t \right]^{\alpha} dv = (1-\alpha) \left(\frac{\alpha}{\chi} \right)^{\frac{\alpha}{1-\alpha}} A_t \quad (5)$$

Real Equilibrium Wages

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- The real equilibrium wage in the case of drastic innovation is expressed as:

$$w_t = (1-\alpha)L_t^{-\alpha}\int_0^1 A_t(v)^{1-\alpha}\left[\alpha^{\frac{1}{1-\alpha}} A_t(v)L_t\right]^{\alpha} dv = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t \qquad (6)$$

- Innovation significantly influences productivity and, consequently, wages and output.
- Non-drastic and drastic innovations yield varying scales of economic outcomes, highlighting the role of innovation size.
- Aggregate productivity is central in determining real wages and GDP growth.

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GDP: Value Added Approach

GDP is the sum of the value added created in the economy over a certain period:

$$GDP_t = \underbrace{Y_t - \int_0^1 p_t(v)x_t(v)dv}_{\text{Final Sector}} + \underbrace{\int_0^1 p_t(v)x_t(v)dv - \int_0^1 m_t(v)dv}_{\text{Intermediate Sector}} \quad (7)$$

where $x_t(v) = m_t(v)$, so:

$$GDP_t = Y_t - \int_0^1 x_t(v)dv \quad (8)$$

- In the case of non-drastic innovations, GDP is given by:

$$GDP_t = Y_t - \int_0^1 x_t(v)dv = \left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{\alpha}{\chi}\right) A_t L_t \quad (9)$$

- In the case of drastic innovations, GDP is given by:

$$GDP_t = Y_t - \int_0^1 x_t(v)dv = (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}} A_t L_t \quad (10)$$

GDP: Income Approach

GDP is the sum of the incomes distributed in the economy over a certain period:

$$GDP_t = w_t L_t + \int_0^1 \Pi_t(v)dv \tag{11}$$

- **Prove that GDP by the income approach equals GDP by the value-added approach.**

The equilibrium GDP per worker growth is given by the growth of the productivity:

$$G_t = \frac{A_{t+1} - A_t}{A_t} \tag{12}$$

Proof.

$\frac{GDP_t}{L_t} = \Gamma A_t$ where $\Gamma = \left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{\alpha}{\chi}\right)$ in the case of non-drastic innovation and $\Gamma = (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}$ in the case of drastic innovation. □

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Innovation Technology (1/2)

The intermediate firm makes its choices in two stages:

- (i) it chooses the level of productivity (i.e., quality) by innovating, then
- (ii) it chooses its level of production and the price it wishes to set.

We consider a simple innovation technology: entrepreneurs in sector v invest the amount $Z_t(v)$ to generate an innovation with probability $\mu_t(v)$. The probability of innovating depends on the amount of resources devoted to innovation:

$$\mu_t(v) = \lambda \left(\frac{Z_t(v)}{A_{t+1}(v)} \right)^{\eta} \tag{13}$$

where $0 < \eta < 1$ and μ measures the productivity of R&D.
The intermediate firm chooses the investment in R&D, $Z_t(v)$, to maximize:

$$\beta \mu_t(v) \Pi_{t+1}(v) - Z_t(v) \tag{14}$$

where β is the discount rate or the rate of time preference.

Innovation Technology (2/2)

The intermediate firm chooses the investment in R&D, $Z_t(v)$, to maximize:

$$\max_{Z_t(v)} \beta\pi\lambda \left(\frac{Z_t(v)}{A_{t+1}(v)}\right)^{\eta} L_{t+1} A_{t+1}(v) - Z_t(v) \tag{15}$$

The first order condition implies :

$$\eta\beta\pi\lambda \left(\frac{Z_t(v)}{A_{t+1}(v)}\right)^{\eta-1} L_{t+1} = 1 \iff \frac{Z_t(v)}{A_{t+1}(v)} = (\eta\beta\pi\lambda L_{t+1})^{\frac{1}{1-\eta}} \tag{16}$$

Investment in R&D is greater in the most advanced sectors. And the equilibrium innovation probability is given by :

$$\mu_t(v) = \lambda\,(\eta\beta\pi\lambda L_{t+1})^{\frac{\eta}{1-\eta}} := \mu_t \tag{17}$$

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- The productivity (quality) of a sector v varies randomly: a sector v innovates with a probability $\mu_t(v)$ and thus increases productivity by an amount γ which measures the size or importance of the innovation (assumed exogenous and identical in all sectors). If the sector does not innovate, the productivity in period $t+1$ remains the same as in the current period, and the firm is randomly replaced by another firm with the same productivity level. In summary, we have:

$$A_{t+1}(v) = \begin{cases} \gamma A_t(v) & \text{with probability } \mu_t \\ A_t(v) & \text{with probability } 1-\mu_t \end{cases} \tag{18}$$

- Productivity is a random process whose mathematical expectation is given by:

$$A_{t+1}(v) = \mu_t \gamma A_t(v) + (1-\mu_t)A_t(v) = A_t(v) + (\gamma - 1)\mu_t A_t(v) \tag{19}$$

- Taking the integral of both sides:

$$A_{t+1} = A_t + (\gamma - 1)\mu_t A_t \tag{20}$$

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Long-Term Economic Growth and Policy

- The GDP per worker (per capita) at steady state is written as:

$$\frac{GDP_t}{L_t} = \Gamma A_t \tag{21}$$

- The growth rate of GDP per capita at steady state is given by:

$$G = \frac{A_{t+1} - A_t}{A_t} = (\gamma - 1)\mu_t \tag{22}$$

- **Implication 1**: The long-term growth rate of the economy is endogenous, meaning that growth is sustainable and does not depend on exogenous technical progress.

- **Implication 2**: Any policy aimed at promoting innovation (such as subsidies to increase the productivity parameter of R&D, λ) increases μ_t and then economic growth.

Scale Effects

▷ **Common Limitation of Endogenous Growth Models**

- Endogenous growth models have a common limitation: the equilibrium growth rate depends positively on the size of the population or the number of researchers employed in R&D (in models where skilled labor is required in research).
- Jones (1995) showed that since 1953 in the United States, the number of researchers and engineers in R&D has increased ninefold without significant productivity gains.

▷ **Solutions to Scale Effects**

- (i) Consider constant population $L = 1$, (ii) use semi-endogenous growth (Jones, 1995), and (iii) use the solution of Young (1998) and Howitt (1999) by introducing both vertical and horizontal innovation.
- Empirical studies by Ha and Howitt (2007), Madsen (2008), and Zachariadis (2003, 2004) support the third approach.
- Kremer (1993) shows that scale effects played a role globally.

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Technological Adoption and Convergence

- Technological transfer implies that countries share the same long-term growth rate.
- However, some technologies developed at the frontier are not suitable for poorer countries (Basu and Weil, 1998; Acemoglu and Zilibotti, 2001).
- Technological transfer can be blocked by local interests (Parente and Prescott, 1994, 1999).
- Some countries adopt institutions that do not allow full benefit from technological transfer (Acemoglu, Aghion, and Zilibotti, 2004).
- Technological transfer requires innovation or investment by the adopting country (Cohen and Levinthal, 1989; Griffith, Redding, and Van Reenen, 2001).
- Human capital increases the "absorption capacity" of technology (Nelson and Phelps, 1966; Benhabib and Spiegel, 1994).
- When convergence occurs, it is explained by the "advantage of backwardness" (Gerschenkron, 1962).

Technology Adoption

- To account for the fact that some countries have no incentive to innovate, we use the following innovation cost function:

$$\frac{Z_t(v)}{A_{t+1}(v)} = \eta \mu_t(v) + \frac{\psi}{2}\mu_t(v)^2 \tag{23}$$

where $\eta, \psi > 0$, $\beta\pi < \eta + \psi$, and $L_t \equiv 1$.

- The innovator's problem is written as:

$$\max_{\mu_t(v)} \beta \mu_t(v) \Pi_{t+1}(v) - Z_t(v) = \left[(\beta\pi - \eta)\mu_t(v) - \frac{\psi}{2}\mu_t(v)^2\right] A_{t+1}(v) \tag{24}$$

$$\text{s.t. } 0 \leq \mu_t(v)$$

Equilibrium Conditions

- Lagrangian and Equilibrium Conditions
- Let ϕ be the Kuhn-Tucker multiplier associated with the non-negativity constraint, the Lagrangian of the problem is written as:

$$\mathscr{L}(\mu_t(v);\phi) = \left[(\beta\pi-\eta)\mu_t(v) - \frac{\psi}{2}\mu_t(v)^2\right]A_{t+1}(v) + \phi\mu_t(v) \tag{25}$$

- The first-order conditions are:

$$\mu_t(v) \geq 0, \quad \beta\pi-\eta-\psi\mu_t(v) = -\frac{\phi}{A_{t+1}(v)} \leq 0, \quad \phi\mu_t(v) = 0 \tag{26}$$

So

$$\mu_t(v) = \begin{cases} 0 & \text{if } \beta\pi-\eta \leq 0 \\ \dfrac{\beta\pi-\eta}{\psi} & \text{if } \beta\pi-\eta > 0 \end{cases} \tag{27}$$

- If $\beta\pi > \eta$, then the reward for innovation is sufficiently high relative to the costs to incentivize entrepreneurs to innovate. Otherwise, there is no innovation.

- In each sector v, there is an individual born in period t who is capable of innovating with a probability $\mu_t(v) = \mu$ identical for all sectors and constant over time.
- The productivity dynamics of sector v is given by:

$$A_{t+1}(v) == \begin{cases} \bar{A}_{t+1} & \text{with probability } \mu \\ A_t(v) & \text{with probability } 1-\mu \end{cases} \tag{28}$$

- where $\bar{A}_{t+1}$ is the global technological frontier whose growth rate is given by $g > 0$.
- Since the probability of innovating is identical for all sectors at equilibrium, we obtain, after integrating both sides:

$$A_{t+1} = \mu \bar{A}_{t+1} + (1-\mu) A_t \tag{29}$$

- where $A_t = \int_0^1 A_t(v) dv$ is the average productivity of the economy.

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Proximity to the Technological Frontier

- The dynamics of average productivity at the frontier can be written as:

$$A_{t+1} - A_t = \mu \left(\bar{A}_{t+1} - A_t \right) \tag{30}$$

Therefore, a country's productivity increases the further it is from the frontier.

- The proximity to the technological frontier of a country's average productivity is defined as:

$$a_t = \frac{A_t}{\bar{A}_t}, \text{ with } a_t \in [0, 1]. \tag{31}$$

- From equation (30), the dynamics of proximity to the frontier is given by:

$$a_{t+1} = \mu + \frac{1-\mu}{1+g} a_t \equiv H(a_t) \tag{32}$$

- This dynamic describes whether the economy converges to the frontier.

Conditional Convergence

- The equilibrium proximity is given by:

$$a^* = \frac{(1+g)\mu}{g+\mu} \tag{33}$$

with $0 < a^* < 1$ and g the growth rate of productivity at the frontier.

Proof.

Setting $a_{t+1} = a_t = a^*$ in the proximity dynamics, we get:

$$a^* = \mu + \frac{1-\mu}{1+g} a^* \tag{34}$$

Solving for a^*, we obtain:

$$a^* = \frac{(1+g)\mu}{g+\mu} \tag{35}$$

It is evident that $a^* > 0$ and $a^* < 1$ because $(1+g)\mu < g+\mu$ which is true since $\mu < 1$ as it is a probability. □

Equilibrium Growth Rate

- All countries with $\beta\pi > \eta$ grow at the same long-term rate given by the growth rate of the technological frontier $G = g$.

Proof.

Since in this case, $\mu > 0$, it is the same for $a^* > 0$, and thus:

$$A_{t+1} = a^* \bar{A}_{t+1} \tag{36}$$

Therefore:

$$\frac{A_{t+1}}{A_t} = \frac{\bar{A}_{t+1}}{\bar{A}_t} = 1 + g \tag{37}$$

Hence:

$$G = \frac{A_{t+1} - A_t}{A_t} = g \tag{38}$$

□

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Convergence to Steady State

- All countries with $\beta\pi > \eta$ converge to their own steady-state level, i.e., $\lim_{t\to\infty}(a_{t+1} - a^*) = 0$.
- Rewriting the proximity dynamics in deviation from its steady state:

$$a_{t+1} - a^* = \mu - a^* + \frac{1-\mu}{1+g} a_t = \mu - a^* + \frac{1-\mu}{1+g}(a_t - a^*) + \frac{1-\mu}{1+g} a^* \tag{39}$$

- Using equation (35) anb by simplifying, we get:

$$a_{t+1} - a^* = \frac{1-\mu}{1+g}(a_t - a^*) \tag{40}$$

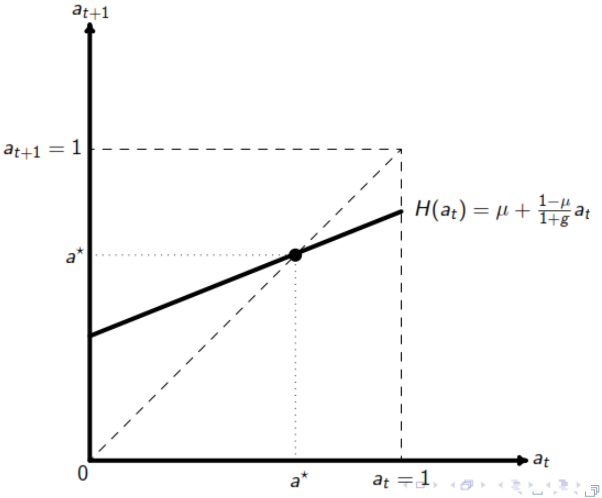
- By recurrence:

$$a_{t+1} - a^* = \left(\frac{1-\mu}{1+g}\right)^{t+1} (a_0 - a^*) \tag{41}$$

- There is convergence because:

$$\lim_{t\to\infty}(a_{t+1} - a^*) = 0 \quad \text{since} \quad \frac{1-\mu}{1+g} < 1 \tag{42}$$

Conditional Convergence



Divergence and Club Convergence

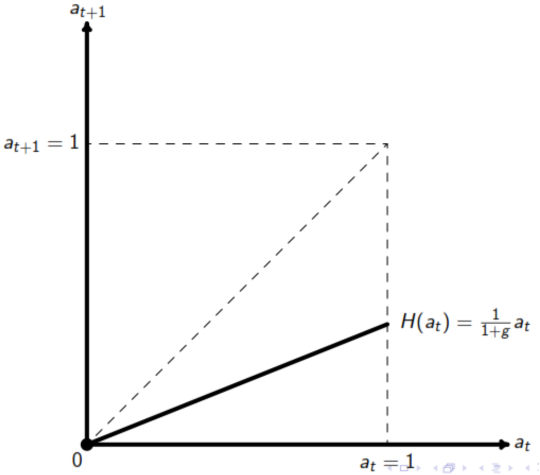
- All countries with $\beta\pi \leq \eta$ stagnate in the long term. In this case, the probability of innovating is zero $\mu = 0$ and thus $a^* = 0$.
- The growth rate G_t is given by:

$$G_t = \frac{A_{t+1}}{A_t} - 1 = (1+g)\frac{a_{t+1}}{a_t} - 1 = (1+g)\frac{H(a_t)}{a_t} - 1 \tag{43}$$

- Using L'Hôpital's rule, we get:

$$\lim_{t\to\infty} G_t = (1+g)\lim_{t\to\infty} H'(a_t) - 1 = (1+g)H'(0) - 1 = 0 \quad \text{since} \quad \mu = 0 \tag{44}$$

Divergence and Convergence Clubs



Readings

- Howitt et Mayer-Foulkes (2004), Journal of Money, Credit and Banking
- Aghion et al. (2005), Quaterly Journal of Economics