ECON 710 - Advanced Macroeconomics

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The Schumpeterian Growth Model

Contents

1 Introdction

The endogenous growth model is a significant theory in economics that attributes long-term economic growth to internal factors within an economy, rather than external influences. Unlike exogenous growth models, which rely on external technological advancements, endogenous growth theory emphasizes the importance of innovation, human capital, and knowledge spillovers. Innovation, driven by research and development (R&D) within the economy, plays a crucial role in this model. Investments in education and training enhance the productivity of the workforce, contributing to sustained economic growth. Additionally, the spread of ideas and innovations, known as knowledge spillovers, benefits other firms and sectors, further driving growth. Policies that encourage innovation, education, and knowledge sharing are therefore essential for achieving sustained economic growth. Prominent examples of endogenous growth models include the AK model and the Uzawa-Lucas model.

The Schumpeterian paradigm, named after the renowned economist Joseph Schumpeter, is a specific type of endogenous growth model that focuses on the role of entrepreneurial innovation and creative destruction in driving economic growth. According to this paradigm, growth is generated by a continuous process of quality-improving innovations. Creative destruction, a key concept in this model, refers to the process by which new innovations replace old technologies, leading to the obsolescence of existing products and processes. This dynamic process is essential for economic progress.

The Schumpeterian model operates under several key assumptions. It assumes the use of labor and vertically differentiated intermediate goods in production. The final goods market is considered perfectly competitive, while the intermediate goods market is monopolistic. Innovation in this model depends heavily on the resources devoted to R&D, which in turn determines the average productivity level. Entrepreneurs play a pivotal role in this paradigm, as they invest in R&D to innovate, motivated by the potential for monopoly rents, which are profits from being the sole provider of a new product or technology.

The Schumpeterian growth theory also examines how market structures, competition, and policies influence innovation and growth. Economists like Philippe Aghion and Peter Howitt have further developed and refined this theory, integrating it into a broader framework for understanding both macroeconomic growth and microeconomic issues related to innovation, which we will expose in this chapter.

2 Goods Production

There are three classes of tradeable objects: labor, a consumption good, and a continuum of intermediate goods $\nu \in [0,1]$. There is also a continuum of identical infinitely-lived individuals, with mass L_t, each endowed with a one-unit flow of labor, and each with identical intertemporally additive preferences defined over lifetime consumption. The final good is produced competitively using labor and a continuum of intermediate goods as inputs, with the aggregate production function given by:

$$Y_t = L_t^{1-\alpha}\left(\int_0^1 A_t(\nu)^{1-\alpha} x_t(\nu)^{\alpha} d\nu\right) \tag{1}$$

where Y_t is the output of final goods, L_t represents labor demand, $x_t(\nu)$ is the flow of intermediate goods of variety ν and quality q, and $A_t(\nu)$ is the highest quality of intermediate goods of variety ν. The mass of intermediate goods is normalized to 1, so $\nu \in [0,1]$. This equation shows how the final output depends on labor and the quality and quantity of intermediate goods used in production.

2.1 Final Goods Producers Profit Maximization

The representative firm in the final goods sector aims to maximize its profit, which is given by:

$$\max_{L_t,[x_t(\nu)]_{\nu\in[0,1]}} \Pi_t = L_t^{1-\alpha}\left(\int_0^1 A_t(\nu)^{1-\alpha} x_t(\nu)^{\alpha} d\nu\right) - \int_0^1 p_t(\nu) x_t(\nu) d\nu - w_t L_t \tag{2}$$

Here, Π_t represents the profit, $p_t(\nu)$ is the price of intermediate goods of variety ν, and w_t is the real wage. The firm chooses the quantity of labor and each variety of intermediate goods to use. The final goods sector is competitive, so the price of the final good (normalized to 1), the price of each variety of intermediate goods $p_t(\nu)$, and the real wage w_t are given.

The first-order conditions for profit maximization with respect to $x_t(\nu)$ and L_t are:

$$\frac{\partial \Pi_t}{\partial x_t(\nu)} = \alpha L_t^{1-\alpha} A_t(\nu)^{1-\alpha} x_t(\nu)^{\alpha-1} - p_t(\nu) = 0 \tag{3}$$

$$\frac{\partial \Pi_t}{\partial L_t} = (1-\alpha) L_t^{-\alpha}\left(\int_0^1 A_t(\nu)^{1-\alpha} x_t(\nu)^{\alpha} d\nu\right) - w_t = 0 \tag{4}$$

These conditions imply that the firm equalizes the marginal productivity of each factor (intermediate goods and labor) to its price. The first equation shows the optimal choice of intermediate goods, while the second equation shows the optimal choice of labor.

2.2 Inverse Demand Function for Intermediate Goods

The representative firm producing the final good is willing to pay up to:

$$p_t(\nu) = \alpha L_t^{1-\alpha} A_t(\nu)^{1-\alpha} x_t(\nu)^{\alpha-1} \tag{5}$$

for intermediate goods of variety ν. The inverse demand function for intermediate goods of variety ν and quality $A_t(\nu)$ is:

$$x_t(\nu) = \left(\frac{\alpha}{1-\alpha}\right) p_t(\nu)^{-\frac{1}{1-\alpha}} L_t A_t(\nu) \tag{6}$$

This equation shows how the quantity demanded of intermediate goods depends on their price, the amount of labor, and their quality. The price elasticity of demand is constant and given by $\frac{1}{1-\alpha}$ in absolute value, indicating how sensitive the quantity demanded is to changes in price.

2.3 Intermediate Goods Production

Each variety ν is produced by a monopoly holding a patent obtained through innovation. The patent and thus the monopoly last for one period. After one period, a new monopoly appears either through innovation or randomly replacing the old monopoly. The intermediate firm makes two choices: (i) quality (innovation) and (ii) quantity produced. Innovation is driven by the prospect of profits, so the firm solves for quantity before solving for quality.

For each variety ν, there is an infinite number of firms in a competitive fringe capable of copying the production plans of existing intermediate goods (reverse engineering). It takes $\chi > 1$ units of final goods to produce one unit of intermediate goods: $m_t^c(\nu) = \chi x_t^c(\nu)$, at a higher cost than the innovator. Innovation is drastic if the monopoly sets its monopoly price, and non-drastic if constrained by the less efficient competitive fringe, practicing a limit pricing strategy: $p_t(\nu) = \chi$. The intermediate monopoly maximizes its profit:

$$\max_{x_t(\nu), p_t(\nu), m_t(\nu)} \Pi_t(\nu) = p_t(\nu) x_t(\nu) - m_t(\nu) \tag{7}$$

$$\text{subject to : } x_t(\nu) = m_t(\nu) \quad \text{(Technological constraint)}$$

$$p_t(\nu) = \begin{cases} \alpha L_t^{1-\alpha} A_t(\nu)^{1-\alpha} x_t(\nu)^{\alpha-1} & \text{(Drastic innovation)} \\ \chi & \text{(Non-drastic innovation)} \end{cases}$$

► **For drastic innovation**, after replacing the price and the cost functions in the objective and derivate on $x_t(\nu)$, the equilibrium quantity produced is:

$$x_t(\nu) = \alpha^{\frac{2}{1-\alpha}} L_t A_t(\nu) \tag{8}$$

where the equilibrium price is:

$$p_t(\nu) = \frac{1}{\alpha} \tag{9}$$

and the equilibrium profit is:

$$\Pi_t(\nu) = \pi L_t A_t(\nu) \tag{10}$$

with $\pi = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}$.

► **For non-drastic innovation**, the equilibrium quantity produced is:

$$x_t(\nu) = \left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t(\nu) L_t \tag{11}$$

where the equilibrium price is:

$$p_t(\nu) = \chi \tag{12}$$

and the equilibrium profit for inovator is :

$$\Pi_t(\nu) = \pi L_t A_t(\nu) \tag{13}$$

with $\pi = (\chi - 1)\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}}$. From Equation (13), the equilibrium profit depends positively on the parameter χ, meaning:

- **Institutional View:** Higher profits (and thus innovation and growth) are linked to stronger property rights protection, such as patents that increase imitation costs for the competitive fringe.

- **Competitive View:** Following the Schumpeterian trade-off, an increase in competition that reduces χ decreases equilibrium profits, lowering innovation incentives. If $\chi = 1$, profits are zero, and there is no incentive to innovate, rendering the model neoclassical.

Proof: The derivative with respect to χ is: $\frac{d\pi}{d\chi} = \alpha^{\frac{1}{1-\alpha}}\left[\chi^{-\frac{1}{1-\alpha}} - \chi^{-1}\right] > 0$ since $\chi < \frac{1}{\alpha}$.

2.4 Equilibrium Final Goods Production

The production of equilibrium final goods in the case of non-drastic innovation is:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_t(\nu)^{1-\alpha} \left[\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t(\nu) L_t\right]^{\alpha} d\nu \tag{14}$$

$$= \left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}} L_t A_t \tag{15}$$

where:

$$A_t = \int_0^1 A_t(\nu) d\nu \tag{16}$$

represents the average productivity of the economy.

In the case of drastic innovation, the equilibrium final goods production is given by:

$$Y_t = L_t^{1-\alpha} \int_0^1 A_t(\nu)^{1-\alpha} \left[\alpha^{\frac{1}{1-\alpha}} A_t(\nu) L_t\right]^\alpha d\nu \tag{17}$$

$$= \alpha^{\frac{2\alpha}{1-\alpha}} L_t A_t \tag{18}$$

2.5 Real Equilibrium Wages

The real equilibrium wage in the case of non-drastic innovation is:

$$w_t = (1-\alpha) L_t^{-\alpha} \int_0^1 A_t(\nu)^{1-\alpha} \left[\left(\frac{\alpha}{\chi}\right)^{\frac{1}{1-\alpha}} A_t(\nu) L_t\right]^\alpha d\nu \tag{19}$$

$$= (1-\alpha) \left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}} A_t \tag{20}$$

The real equilibrium wage in the case of drastic innovation is expressed as:

$$w_t = (1-\alpha) L_t^{-\alpha} \int_0^1 A_t(\nu)^{1-\alpha} \left[\alpha^{\frac{1}{1-\alpha}} A_t(\nu) L_t\right]^\alpha d\nu \tag{21}$$

$$= (1-\alpha) \alpha^{\frac{2\alpha}{1-\alpha}} A_t \tag{22}$$

Innovation significantly influences productivity and, consequently, wages and output. Non-drastic and drastic innovations yield varying scales of economic outcomes, highlighting the role of innovation size. Aggregate productivity is central in determining real wages and GDP growth.

2.6 Equilibrium Income

GDP: Value Added Approach. GDP is the sum of the value added created in the economy over a certain period:

$$GDP_t = \underbrace{Y_t - \int_0^1 p_t(\nu) x_t(\nu) d\nu}_{\text{Final Sector}} + \underbrace{\int_0^1 p_t(\nu) x_t(\nu) d\nu - \int_0^1 m_t(\nu) d\nu}_{\text{Intermediate Sector}} \tag{23}$$

where $x_t(\nu) = m_t(\nu)$, so:

$$GDP_t = Y_t - \int_0^1 x_t(\nu) d\nu \tag{24}$$

In the case of non-drastic innovations, GDP is given by:

$$GDP_t = Y_t - \int_0^1 x_t(\nu)d\nu \tag{25}$$
$$= \left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{\alpha}{\chi}\right) A_t L_t \tag{26}$$

In the case of drastic innovations, GDP is given by:

$$GDP_t = Y_t - \int_0^1 x_t(\nu)d\nu \tag{27}$$
$$= (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}} A_t L_t \tag{28}$$

GDP: Income Approach. GDP is the sum of the incomes distributed in the economy over a certain period:

$$GDP_t = w_t L_t + \int_0^1 \Pi_t(\nu)d\nu \tag{29}$$

Exercise : Prove that GDP by the income approach equals GDP by the value-added approach. The equilibrium GDP per worker growth is given by the growth of the productivity:

$$G_t = \frac{A_{t+1} - A_t}{A_t} \tag{30}$$

Proof. $\frac{GDP_t}{L_t} = \Gamma A_t$ where $\Gamma = \left(\frac{\alpha}{\chi}\right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{\alpha}{\chi}\right)$ in the case of non-drastic innovation and $\Gamma = (1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}$ in the case of drastic innovation. □

3 Innovation and Growth

3.1 Innovation Technology

The intermediate firm makes its choices in two stages:

- First, it chooses the level of productivity (i.e., quality) by innovating.
- Second, it chooses its level of production and the price it wishes to set.

We consider a simple innovation technology: entrepreneurs in sector ν invest the amount $Z_t(\nu)$ to generate an innovation with probability $\mu_t(\nu)$. The probability of innovating depends on the amount of resources devoted to innovation:

$$\mu_t(\nu) = \lambda \left(\frac{Z_t(\nu)}{A_{t+1}(\nu)}\right)^{\eta} \tag{31}$$

where $0 < \eta < 1$ and λ measures the productivity of R&D. The intermediate firm chooses the investment in R&D, $Z_t(\nu)$, to maximize:

$$\beta \mu_t(\nu)\Pi_{t+1}(\nu) - Z_t(\nu) \tag{32}$$

where β is the discount rate or the rate of time preference.

The intermediate firm chooses the investment in R&D, $Z_t(\nu)$, to maximize:

$$\max_{Z_t(\nu)} \beta\pi\lambda \left(\frac{Z_t(\nu)}{A_{t+1}(\nu)}\right)^{\eta} L_{t+1}A_{t+1}(\nu) - Z_t(\nu) \tag{33}$$

The first order condition implies:

$$\eta\beta\pi\lambda \left(\frac{Z_t(\nu)}{A_{t+1}(\nu)}\right)^{\eta-1} L_{t+1} = 1 \Longleftrightarrow \frac{Z_t(\nu)}{A_{t+1}(\nu)} = (\eta\beta\pi\lambda L_{t+1})^{\frac{1}{1-\eta}} \tag{34}$$

Investment in R&D is greater in the most advanced sectors. The equilibrium innovation probability is given by:

$$\mu_t(\nu) = \lambda\,(\eta\beta\pi\lambda L_{t+1})^{\frac{\eta}{1-\eta}} := \mu_t \tag{35}$$

3.2 Productivity Dynamics and Economic Growth

The productivity (quality) of a sector ν varies randomly: a sector ν innovates with a probability $\mu_t(\nu)$ and thus increases productivity by an amount γ which measures the size or importance of the innovation (assumed exogenous and identical in all sectors). If the sector does not innovate, the productivity in period $t+1$ remains the same as in the current period, and the firm is randomly replaced by another firm with the same productivity level. In summary, we have:

$$A_{t+1}(\nu) = \begin{cases} \gamma A_t(\nu) & \text{with probability } \mu_t \\ A_t(\nu) & \text{with probability } 1-\mu_t \end{cases} \tag{36}$$

Productivity is a random process whose mathematical expectation is given by:

$$A_{t+1}(\nu) = \mu_t\gamma A_t(\nu) + (1-\mu_t)A_t(\nu) = A_t(\nu) + (\gamma-1)\mu_t A_t(\nu) \tag{37}$$

Taking the integral of both sides:

$$A_{t+1} = A_t + (\gamma-1)\mu_t A_t \tag{38}$$

The GDP per worker (per capita) at steady state is written as:

$$\frac{GDP_t}{L_t} = \Gamma A_t \tag{39}$$

The growth rate of GDP per capita at steady state is given by:

$$G = \frac{A_{t+1} - A_t}{A_t} = (\gamma - 1)\mu_t \tag{40}$$

Implication 1: The long-term growth rate of the economy is endogenous, meaning that growth is sustainable and does not depend on exogenous technical progress.
Implication 2: Any policy aimed at promoting innovation (such as subsidies to increase the productivity parameter of R&D, λ) increases μ_t and then economic growth.

4 Technological Adoption and Convergence

Technological transfer implies that countries share the same long-term growth rate. However, some technologies developed at the frontier are not suitable for poorer countries (Basu and Weil, 1998; Acemoglu and Zilibotti, 2001).

Technological transfer can be blocked by local interests (Parente and Prescott, 1994, 1999). Some countries adopt institutions that do not allow full benefit from technological transfer (Acemoglu, Aghion, and Zilibotti, 2004). Technological transfer requires innovation or investment by the adopting country (Cohen and Levinthal, 1989; Griffith, Redding, and Van Reenen, 2001). Human capital increases the "absorption capacity" of technology (Nelson and Phelps, 1966; Benhabib and Spiegel, 1994). When convergence occurs, it is explained by the "advantage of backwardness" (Gerschenkron, 1962).

4.1 Technology Adoption

To account for the fact that some countries have no incentive to innovate, we use the following innovation cost function:

$$\frac{Z_t(\nu)}{A_{t+1}(\nu)} = \eta \mu_t(\nu) + \frac{\psi}{2}\mu_t(\nu)^2 \tag{41}$$

where $\eta, \psi > 0$, $\beta\pi < \eta + \psi$, and $L_t \equiv 1$. The innovator's problem is written as:

$$\max_{\mu_t(\nu)} \beta\mu_t(\nu)\Pi_{t+1}(\nu) - Z_t(\nu) = \left[(\beta\pi - \eta)\mu_t(\nu) - \frac{\psi}{2}\mu_t(\nu)^2\right] A_{t+1}(\nu) \tag{42}$$

s.t. $\mu_t(\nu) \geq 0$.

Let ϕ be the Kuhn-Tucker multiplier associated with the non-negativity constraint. The

Lagrangian of the problem is written as:

$$\mathscr{L}(\mu_t(v);\phi) = \left[(\beta\pi - \eta)\mu_t(v) - \frac{\psi}{2}\mu_t(v)^2\right] A_{t+1}(v) + \phi\mu_t(v) \tag{43}$$

The first-order conditions are:

$$\mu_t(v) \geq 0, \quad \beta\pi - \eta - \psi\mu_t(v) = -\frac{\phi}{A_{t+1}(v)} \leq 0, \quad \phi\mu_t(v) = 0 \tag{44}$$

So,

$$\mu_t(v) = \begin{cases} 0 & \text{if } \beta\pi - \eta \leq 0 \\ \dfrac{\beta\pi - \eta}{\psi} & \text{if } \beta\pi - \eta > 0 \end{cases} \tag{45}$$

If $\beta\pi > \eta$, then the reward for innovation is sufficiently high relative to the costs to incentivize entrepreneurs to innovate. Otherwise, there is no innovation.

4.2 Productivity Dynamics

In each sector v, there is an individual born in period t who is capable of innovating with a probability $\mu_t(v) = \mu$ identical for all sectors and constant over time.

The productivity dynamics of sector v is given by:

$$A_{t+1}(v) = \begin{cases} \bar{A}_{t+1} & \text{with probability } \mu \\ A_t(v) & \text{with probability } 1-\mu \end{cases} \tag{46}$$

where $\bar{A}_{t+1}$ is the global technological frontier whose growth rate is given by $g > 0$.

Since the probability of innovating is identical for all sectors at equilibrium, we obtain, after integrating both sides:

$$A_{t+1} = \mu\bar{A}_{t+1} + (1-\mu)A_t \tag{47}$$

where $A_t = \int_0^1 A_t(v)dv$ is the average productivity of the economy.

The dynamics of average productivity at the frontier can be written as:

$$A_{t+1} - A_t = \mu\left(\bar{A}_{t+1} - A_t\right) \tag{48}$$

Therefore, a country's productivity increases the further it is from the frontier. The proximity to the technological frontier of a country's average productivity is defined as:

$$a_t = \frac{A_t}{\bar{A}_t}, \text{ with } a_t \in [0,1]. \tag{49}$$

From equation (48), the dynamics of proximity to the frontier is given by:

$$a_{t+1} = \mu + \frac{1-\mu}{1+g} a_t \equiv H(a_t) \tag{50}$$

This dynamic describes whether the economy converges to the frontier.

4.3 Conditional Convergence

The equilibrium proximity is given by:

$$a^* = \frac{(1+g)\mu}{g+\mu} \tag{51}$$

with $0 < a^* < 1$ and g the growth rate of productivity at the frontier.

Proof. Setting $a_{t+1} = a_t = a^*$ in the proximity dynamics, we get:

$$a^* = \mu + \frac{1-\mu}{1+g} a^* \tag{52}$$

Solving for a^*, we obtain:

$$a^* = \frac{(1+g)\mu}{g+\mu} \tag{53}$$

It is evident that $a^* > 0$ and $a^* < 1$ because $(1+g)\mu < g+\mu$ which is true since $\mu < 1$ as it is a probability. □

All countries with $\beta\pi > \eta$ grow at the same long-term rate given by the growth rate of the technological frontier $G = g$.

Proof. Since in this case, $\mu > 0$, it is the same for $a^* > 0$, and thus:

$$A_{t+1} = a^* \bar{A}_{t+1} \tag{54}$$

Therefore:

$$\frac{A_{t+1}}{A_t} = \frac{\bar{A}_{t+1}}{\bar{A}_t} = 1+g \tag{55}$$

Hence:

$$G = \frac{A_{t+1} - A_t}{A_t} = g \tag{56}$$

□

All countries with $\beta\pi > \eta$ converge to their own steady-state level, i.e., $\lim_{t\to\infty}(a_{t+1} - a^*) = 0$.

Rewriting the proximity dynamics in deviation from its steady state:

$$a_{t+1} - a^* = \mu - a^* + \frac{1-\mu}{1+g} a_t = \mu - a^* + \frac{1-\mu}{1+g}(a_t - a^*) + \frac{1-\mu}{1+g} a^* \tag{57}$$

Using equation (53) and by simplifying, we get:

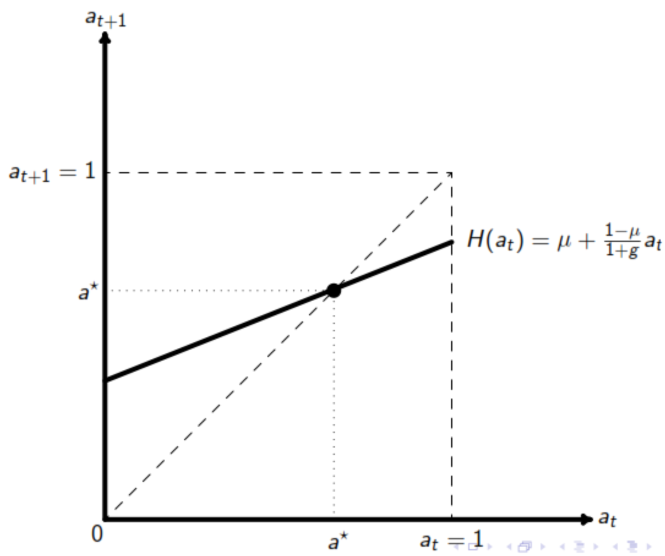
$$a_{t+1} - a^* = \frac{1-\mu}{1+g}(a_t - a^*) \tag{58}$$

By recurrence:

$$a_{t+1} - a^* = \left(\frac{1-\mu}{1+g}\right)^{t+1} (a_0 - a^*) \tag{59}$$

There is convergence because:

$$\lim_{t\to\infty}(a_{t+1} - a^*) = 0 \quad \text{since} \quad \frac{1-\mu}{1+g} < 1 \tag{60}$$



4.4 Divergence and Club Convergence

All countries with $\beta\pi \leq \eta$ stagnate in the long term. In this case, the probability of innovating is zero $\mu = 0$ and thus $a^* = 0$. The growth rate G_t is given by:

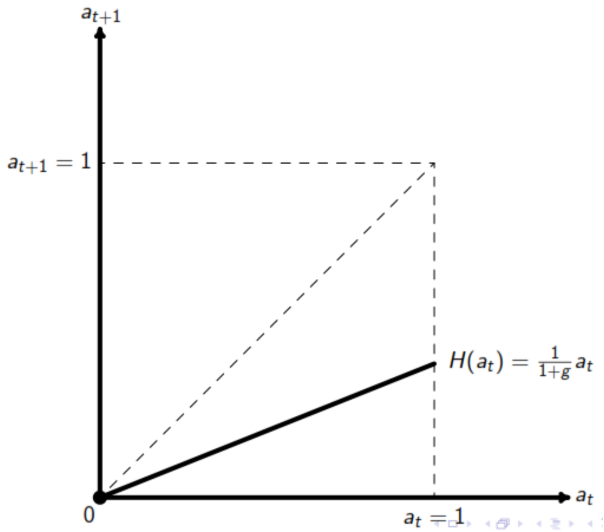
$$G_t = \frac{A_{t+1}}{A_t} - 1 = (1+g)\frac{a_{t+1}}{a_t} - 1 = (1+g)\frac{H(a_t)}{a_t} - 1 \tag{61}$$

Using L'Hôpital's rule, we get:

$$\lim_{t\to\infty} G_t = (1+g)\lim_{t\to\infty} H'(a_t) - 1 = (1+g)H'(0) - 1 = 0 \quad \text{since} \quad \mu = 0 \tag{62}$$

Readings

- Howitt et Mayer-Foulkes (2004), Journal of Money, Credit and Banking



- Aghion et al. (2005), Quarterly Journal of Economics